Ma 541: Topics in Applied Mathematics: Wavelet Algorithms
Homework Assignment 2

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Due Friday, October 3rd, 2008

1. Suppose that ϕ is the scaling function for an orthogonal MRA with two-scale equation

$$\phi(t) = \sum_k h(k)\sqrt{2}\phi(2t-k).$$

   Fix an integer M and define $g(k) \stackrel{\text{def}}{=} (-1)^k \bar{h}(2M+1-k)$. Show that

   (a) $\displaystyle\sum_{k\in\mathbf{Z}} \bar{g}(k+2m)g(k+2n) = \delta(m-n), \qquad m,n \in \mathbf{Z};$

   (b) $\displaystyle\sum_{k\in\mathbf{Z}} \bar{g}(k+2m)h(k+2n) = 0, \qquad m,n \in \mathbf{Z};$

   (c) $\displaystyle\sum_{k\in\mathbf{Z}} \Big(\bar{g}(m+2k)g(n+2k) + \bar{h}(m+2k)h(n+2k)\Big) = \delta(m-n), \qquad n,m \in \mathbf{Z};$

2. Suppose that h is the filter sequence for an orthogonal MRA. Let $[a,b] \subset \mathbf{Z}$ be the shortest interval satisfying ($k < a$ or $k > b$) implies $h(k) = 0$. Prove that $b-a$ must be odd. (Thus the support length $|[a,b]| = 1+b-a$ of an orthogonal MRA filter must be even.)

3. Implement periodized convolution/decimation and its adjoint. Assume that the input signal has even integer period $2M$ and that the filter sequence is supported in the index interval $[0, 2L-1]$ with $L \leq M$.

4. Prepare a table of coefficients for the first 5 Daubechies orthogonal wavelet filters, both h and g. Insure 24 digits of accuracy.

5. Plot the first 5 Daubechies orthogonal wavelets and scaling functions, sampled at 256 points within their support.