1. Suppose that $\phi$ is the scaling function for an orthogonal MRA with two-scale equation

$$\phi(t) = \sum_k h(k) \sqrt{2} \phi(2t - k).$$

Fix an integer $M$ and define $g(k) \overset{\text{def}}{=} (-1)^k \bar{h}(2M + 1 - k)$. Show that

(a) $\sum_{k \in \mathbb{Z}} \bar{g}(k + 2m) g(k + 2n) = \delta(m - n), \quad m, n \in \mathbb{Z};$

(b) $\sum_{k \in \mathbb{Z}} \bar{g}(k + 2m) h(k + 2n) = 0, \quad m, n \in \mathbb{Z};$

(c) $\sum_{k \in \mathbb{Z}} \left( \bar{g}(m + 2k) g(n + 2k) + \bar{h}(m + 2k) h(n + 2k) \right) = \delta(m - n), \quad n, m \in \mathbb{Z};$

2. Suppose that $h$ is the filter sequence for an orthogonal MRA. Let $[a, b] \subset \mathbb{Z}$ be the shortest interval satisfying ($k < a$ or $k > b$) implies $h(k) = 0$. Prove that $b - a$ must be odd. (Thus the support length $|[a, b]| = 1 + b - a$ of an orthogonal MRA filter must be even.)

3. Implement periodized convolution/decimation and its adjoint. Assume that the input signal has even integer period $2M$ and that the filter sequence is supported in the index interval $[0, 2L - 1]$ with $L \leq M$.

4. Prepare a table of coefficients for the first 5 Daubechies orthogonal wavelet filters, both $h$ and $g$. Insure 24 digits of accuracy.

5. Plot the first 5 Daubechies orthogonal wavelets and scaling functions, sampled at 256 points within their support.