



Figure A.5: Huffman code solving Exercise 8.

4. If there are fewer than 10 orphan vertices left, connect them to a common parent, call it the root, and terminate. Otherwise, return to step 2. \square
8. **Solution:** A Huffman code for this combination of alphabet and occurrence probabilities is shown in Figure A.5. It has codeword lengths $n = (2, 4, 4, 3, 2, 4, 3, 4)$, and its bit rate is $\sum_{x \in A} p(x)n(x) = 2.73$, whereas $H(p) = \sum_{x \in A} p(x) \log_2 1/p(x) = 2.69$. \square
9. **Solution:** First count the occurrences of each of the 20 letters in this 85-letter message: 1kpw, 2bsv, 3acd, 4fhly, 5nou, 7l, 8t, 9r, 12e. One Huffman tree for this set of occurrence probabilities is obtained in these steps:

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1k 1p 1w 2b 2s 2v 3a 3c 3d 4f 4h 4i 4y 5n 5o 5u 7l 8t 9r 12e
1w 2kp 2b 2s 2v 3a 3c 3d 4f 4h 4i 4y 5n 5o 5u 7l 8t 9r 12e
2b 2s 2v 3kpw 3a 3c 3d 4f 4h 4i 4y 5n 5o 5u 7l 8t 9r 12e
2v 3kpw 3a 3c 3d 4bs 4f 4h 4i 4y 5n 5o 5u 7l 8t 9r 12e
3a 3c 3d 4bs 4f 4h 4i 4y 5vkpw 5n 5o 5u 7l 8t 9r 12e
3d 4bs 4f 4h 4i 4y 5vkpw 5n 5o 5u 6ac 7l 8t 9r 12e
4f 4h 4i 4y 5vkpw 5n 5o 5u 6ac 7dbs 7l 8t 9r 12e
4i 4y 5vkpw 5n 5o 5u 6ac 7dbs 7l 8fh 8t 9r 12e
5vkpw 5n 5o 5u 6ac 7dbs 7l 8iy 8fh 8t 9r 12e
5o 5u 6ac 7dbs 7l 8iy 8fh 8t 9r 10vkpwn 12e
6ac 7dbs 7l 8iy 8fh 8t 9r 10ou 10vkpwn 12e
7l 8iy 8fh 8t 9r 10ou 10vkpwn 12e 13acdb
8fh 8t 9r 10ou 10vkpwn 12e 13acdb 15liy
9r 10ou 10vkpwn 12e 13acdb 15liy 16fht
10vkpwn 12e 13acdb 15liy 16fht 19rou
13acdb 15liy 16fht 19rou 22vkpwne
16fht 19rou 22vkpwne 28acdbslly
22vkpwne 28acdbslly 35fhtrou
35fhtrou 50vkpwneacdbslly
85fhtrouvkpwneacdbslly

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These nodes may be rearranged into a tree with its root at the top: