Math 109 Homework 2, Fall 2022 (Answer Key)

1) a) 1/4 of a beat
b) 1 1/2 (or 3/2) beats
c) 31/32 of a beat

2) a) 2/3 of a beat
b) 1/3 of a beat
c) 5/12 of a beat

3) Let $m \geq 0$ and $r \in \mathbb{R}$ and $r \neq 1$

Show $$1 + r + r^2 + \cdots + r^m = \frac{1 - r^{m+1}}{1 - r}$$

Multiplying both sides by $(1-r)$:

$$(1-r)(1 + r + r^2 + \cdots + r^m) = 1 - r^{m+1}$$

$$(1 + r + r^2 + \cdots + r^m) - (r + r^2 + r^3 + \cdots + r^{m+1}) = 1 - r^{m+1}$$

Notice all terms $r, r^2, \ldots, r^m$ cancel

Thus, $1 - r^{m+1} = 1 - r^{m+1}$ ✓

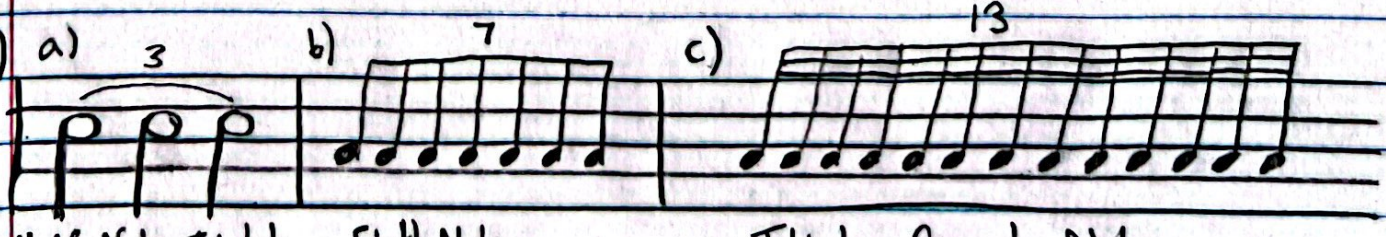
Take $r = 1/2$. Then, assuming your original, undotted note is 1 beat, a note with m dots will have the value outlined by this formula

If your undotted note is n beats,

$$n\left(\frac{1 - r^{m+1}}{1 - r}\right)$$

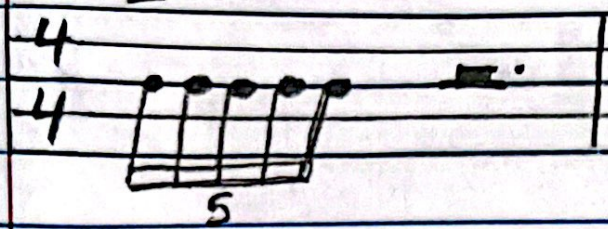
will give the value you need.

4)

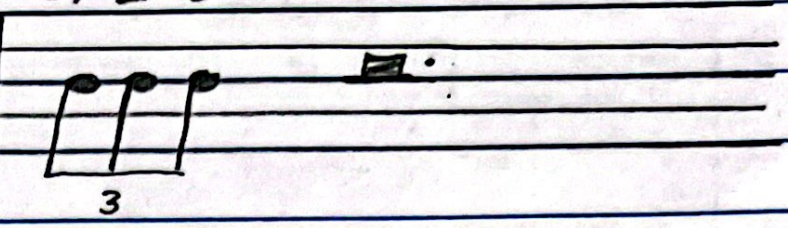


a) Half Note Triplet
b) Eighth Note 7-Tuplet
c) Thirty-Second Note 13-Tuplet

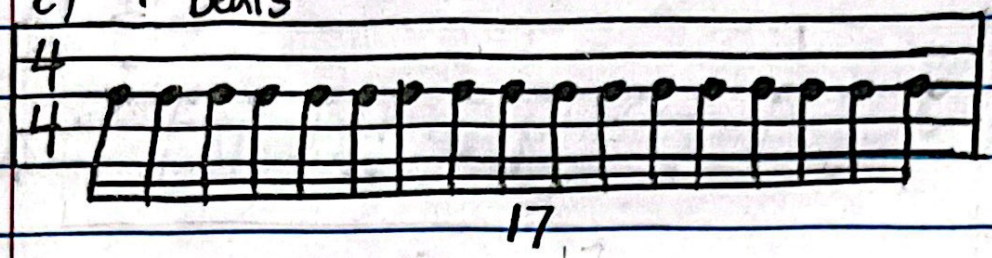
5) a) 1 Beat



b) 1 Beat



c) 4 Beats



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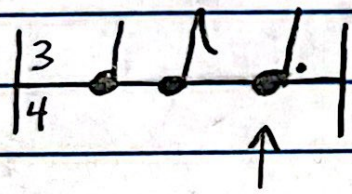
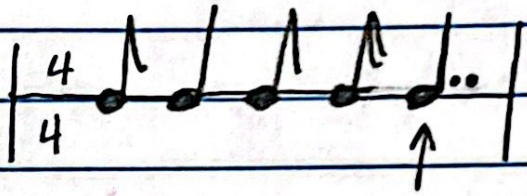
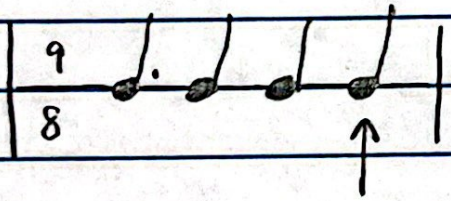
6) a)



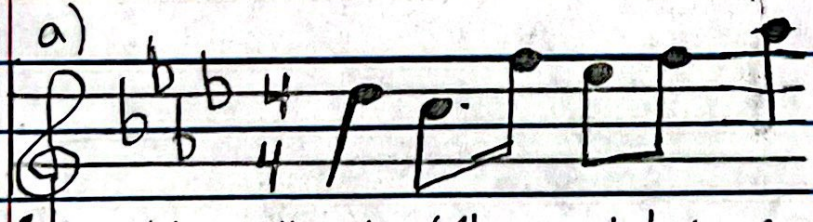
b)



c)

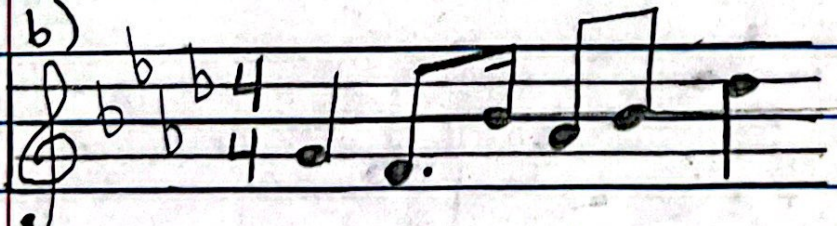


7) a)

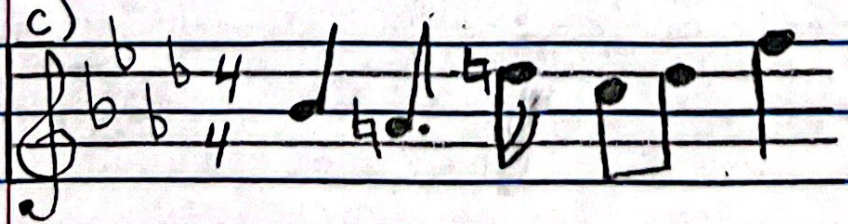


Notice, this is <u>both</u> a diatonic & chromatic transposition

b)



c)



8) **Possible Answers**

Form: ABAC

Transformations

Translations: mm 1 + mm 2

mm 1-4 + mm 5-8

(more options available)

Transposition: Diatonic mm 1-4, mm 5-8

(more options available)

Full Credit given for:

- Identifying the Form
- Identifying two transformations

9) Ionian - Major

Dorian - Minor (m)

Phrygian - Minor (m)

Lydian - Major

Myxolydian - Major

Aeolian - Minor (m)

Locrian - Diminished (dim or °)

10) 1) $\equiv$ on $\mathbb{Z}$ defined by $k \equiv \ell \iff 12 \mid (k-\ell)$

2) $\sim$ on $\mathbb{Z}$ defind by $k \sim \ell \iff$ the musical intervals k semitones and ℓ semitones differ by a multi-octave

In music, there are $n = 12$ equivalence classes and the set is denoted as $\mathbb{Z}_{12}$. Equivalence modulo 12 is octave identification: intervals k and ℓ semitones are equivalent modulo octave if and only if $k \equiv \ell \mod 12$. By moving 12 semitones from a note, we come to another note in the same note class an octave above or below. Thus, ① is the same equivalence relation as ②

~See book pg. 31-32 for more discussion