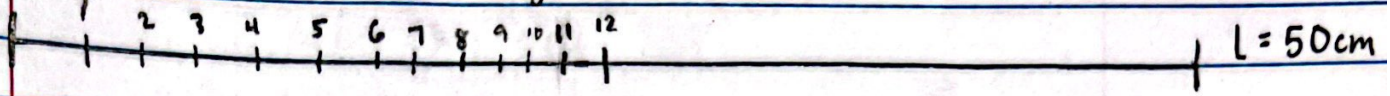
Math 109 HW4 Answer Key Fall 2022

1)



Distances measured from the right end, distances from the left equal $L - d$

1: 47.19 cm
2: 44.54 cm
3: 42.04 cm
4: 39.69 cm
5: 37.46 cm
6: 35.36 cm
7: 33.37 cm
8: 31.50 cm
9: 29.73 cm
10: 28.06 cm
11: 26.49 cm
12: 25.00 cm

* Above not drawn to scale; notice distances between semitones get smaller as you ascend.

2) a) $2^{-4/12} \cdot 100 = \boxed{79.37 \text{ cm}}$

b) $\frac{4}{5} \cdot 100 = \boxed{80 \text{ cm}}$

3) a) $\log_{10}(0.001) = \log_{10}(10^{-3}) = -3\log_{10}(10) = \boxed{-3}$

b) $\log_5 3125 = \log_5(5^5) = 5\log_5(5) = \boxed{5}$

c) $\log_3(\sqrt[10]{81}) = \log_3(81^{1/10}) = \log_3(3^{4/10}) = 4/10 \log_3(3) = \boxed{2/5}$

d) $\log_c(1/\sqrt[n]{c^2}) = \log_c(c^{-2/n}) = -2/n \log_c(c) = \boxed{-2/n}$

e) $\boxed{\log_4(210)}$

f) $\boxed{\log_9(3/8)}$

g) $\log_2 13 + \log_2\sqrt{21} = \boxed{\log_2(13\sqrt{21})}$

h) $\log_c x^4 - \log_{\sqrt{c}} x^{1/2} = \log_c x^4 - \log_c x = \boxed{\log_c(x^3)}$

4) a) $12 \log_2(5) = \boxed{27.86}$

b) $12 \log_2(0.4) = \boxed{-15.86}$

c) $12 \log_2(7/3) = \boxed{14.67}$

d) $12 \log_2(\sqrt[4]{6}) = \boxed{7.75}$

e) $12 \log_2(e) = \boxed{17.31}$

f) $1200 \log_2(0.7) = \boxed{-617.49} \approx \boxed{-617}$

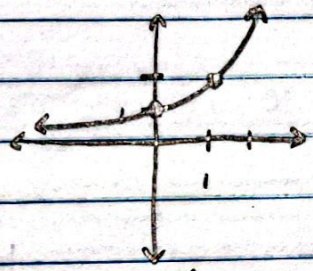
g) $1200 \log_2(3.1) = \boxed{1958.72} \approx \boxed{1959}$

h) $1200 \log_2(3/11) = \boxed{-2249.36} \approx \boxed{-2249}$

i) $1200 \log_2(8/7) = \boxed{231.17} \approx \boxed{231}$

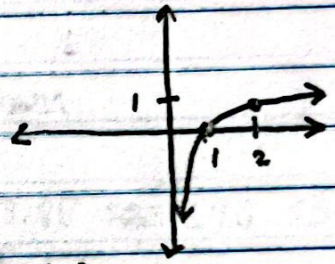
j) $1200 \log_2(\pi/2) = \boxed{781.80} \approx \boxed{782}$

5) a)



$f(x) = 2^x$

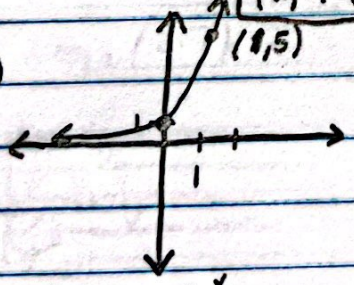
b)



$g(x) = \log_2 x$

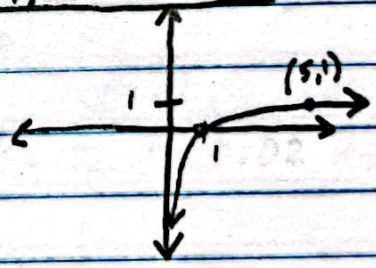
(a) + (b) $(f(x)$ & $g(x))$ are inverses

c)



$r(x) = 5^x$

d)



$s(x) = \log_5 x$

(c) & (d) $(r(x)$ & $s(x))$ are inverses

$\Rightarrow$ $g(x)$ is a vertical stretch of $s(x)$ by a factor of $\log_2(5)$

$g(x) = \log_2(5) \cdot s(x)$

$\log_5(x) = \frac{\log_2 x}{\log_2 5} \Rightarrow \log_2 x = \log_2(5) \cdot \log_5(x)$

$\Rightarrow$ $f(x)$ is a horizontal compression of $r(x)$ by a factor of $\log_5(2)$

$2^x = (5^{\log_5 2})^x$

$f(x) = r(\log_5 2 \cdot x)$

6) Assuming (L1)

$\log_b\left(\frac{x}{y}\right) = \log_b x + \log_b \frac{1}{y}$

$= \log_b x + \log_b y^{-1}$

$= \log_b x - \log_b y$ ✓

Let $s = \log_b x$, $t = \log_b y$

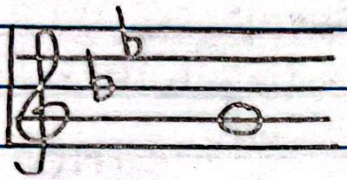
$b^s = x$, $b^t = y$

$x/y = b^s/b^t = b^{s-t}$

$\log_b\left(\frac{x}{y}\right) = \log_b b^{s-t} = s - t$

$= \log_b x - \log_b y$ ✓

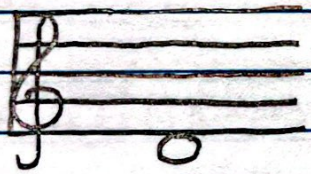
7) a) $12 \log_2 5 \approx 28$ semitones up



$5 \times 77.78 = 388.9$ Hz

G_4

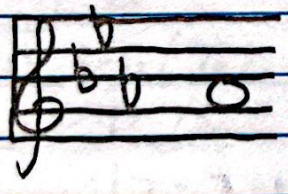
b) $12 \log_2 3/5 \approx 9$ semitones down



$3/5 \times 493.88 \approx 296.33$ Hz

D_4

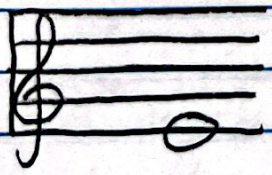
c) $\log_2(2.7) \approx 17$ semitones up



$2.7 \times 155.56 \approx 420.02$ Hz

$A^{\flat}_4$

d) $\log_2(2/\pi) \approx 8$ semitones down



$2/\pi \cdot 523.25 \approx 333.11$ Hz

E_4

8) a) $7/4 = 2^{x/17} \Rightarrow x = 17 \log_2 7/4 = \boxed{13.73}$

b) $3 = 2^{x/7} \Rightarrow x = 7 \log_2(3) = \boxed{11.09}$

c) $0.54 = 2^{x/13} \Rightarrow x = 13 \log_2(0.54) = \boxed{-11.56}$

d) $e = 2^{x/3} \Rightarrow x = 3 \log_2(e) = \boxed{4.33}$

* Answers can slightly fluctuate depending on where rounding happens

9) $n = 11$

$11 \log_2(3/2) \approx 6.43$

6 11-chromatic units

Error: $|6.43 - 6| = 0.43$

$0.43 \cdot \frac{1200}{11} \approx$ 46.91 cents off ≈ 47

$n = 19$

$19 \log_2(3/2) \approx 11.11$

11 19-chromatic units

Error: $|11.11 - 11| = 0.11$

$0.11 \cdot \frac{1200}{19} \approx$ 6.98 cents off ≈ 7

$n = 23$

$23 \log_2(3/2) \approx 13.45$

13 23-chromatic units

Error: $|13 - 13.45| = 0.45$

$0.45 \cdot \left(\frac{1200}{23}\right) \approx$ 23.48 cents off ≈ 23 (or 24, rounding)

$n = 12$

$12 \log_2(3/2) \approx 7.02$

7 12-chromatic units

Error: $|7 - 7.02| = 0.02$

$0.02 \cdot \frac{1200}{12} =$ 2 cents off

$n = 19$ is the best approx. of $n = 11, 19, 23$

12-chromatic is still best.

10 "Moonlight Sonata" 1st Movement

Measure 1: $C^{\#}m$, Im (i)

Measure 2: $C^{\#}m^7$, Im^7 (i^7)

Measure 3(a): A, VI

Measure 3(b): D, $\flat$II

Measure 4(a): $G^{\#7}$, V^7

Measure 4(b): $C^{\#}m$, Im (i)

Measure 5: $C^{\#}m$, Im (i)