EXAM I
Math 109 / Music 109A, Spring 2009

Name _______________________________ Id __________________

Each problem is worth 10 points.

1. Aural: Notate the rhythm (one measure each).

   (a) \[ \frac{4}{4} \]          (b) \[ \frac{12}{8} \]

   Circle the triad type.

   (c) major          (d) minor
       minor          minor
       diminished    diminished

2. Sketch the graphs of these functions by starting with a more basic
   function and applying one or more geometric transformations (shifts or
   stretches). Use the space on page 4 if you need it.

   (a) \[ f(x) = \frac{1}{2} x^2 \]          (b) \[ g(x) = -1 + \sin 2x \]

3. For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose
   existence is asserted in the division algorithm \( n = qm + r \):

   (a) 11, -101 ;

   (b) 5, 3035\( \ell \) + 9, where \( \ell \) some integer.
4. Write the indicated note as a whole note on the given staff, choosing and notating an appropriate clef.

(a) \[ \boxed{\text{G}_2} \]  
(b) \[ \boxed{\text{C}^\#_5} \]  
(c) \[ \boxed{\text{B}^\flat_3} \]

5. For the set \( \{(a, b) \in \mathbb{Z}^2 | b \neq 0\} \) show that the relation \( \sim \) defined by \( (a, b) \sim (a', b') \) if and only if \( ab' - a'b = 0 \) is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers \( \mathbb{Q} \).

OR

For the set \( \mathbb{Z} \) and a fixed positive integer \( m \), show that the relation \( \equiv \) defined by \( k \equiv \ell \) if and only if \( m \mid (k - \ell) \) is an equivalence relation. Explain why there are exactly \( m \) equivalence classes.
6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Lydian with tonic D

(c) Aeolian with tonic B♭

7. Add the needed sharps or flats to notes so that the following gives the Locrian scale tones 1 to 8, from D to D. (Do not alter D.)

8. Extend the following melody with two measures having the same rhythm, employing the following transformations. Do not write in a key change.

(a) diatonic up one scale tone in the second measure

(b) chromatic up a major third (from the original) in the third measure

9. Give the total duration in beats of:

(a) a doubly-dotted half note in $\frac{4}{4}$ time.

(b) a half note in $\frac{9}{8}$ time (compound time signature).

(c) a sixteenth note 9-tuplet in $\frac{4}{4}$ time.
10. For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

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Mary had a little lamb, little lamb, little lamb,
Mary had a little lamb, his fleece was white as snow.
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