EXAM I

Math 109 / Music 109A, Fall 2020

Name _____Solutions_____ Id _____

Each problem is worth 10 points.
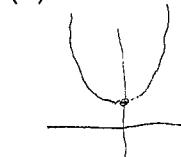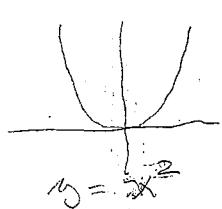
1. <u>Aural</u>: Circle the triad type.

    (a) (major)   (b) major
          minor   (minor)
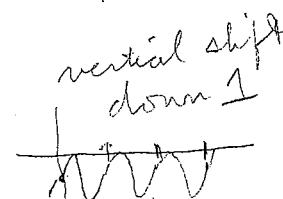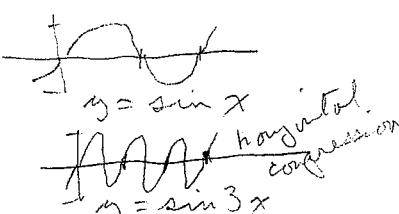
2. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

    (a) $f(x) = x^2 + 1$ (b) $g(x) = -1 + \sin 3x$

    vertical shift down 1

    $y = x^2$
    $y = x^2 + 1$
    vertical shift up 1

    $y = \sin x$
    horizontal compression
    $y = \sin 3x$
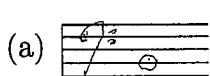
3. For the following pairs of integers m, n, find the numbers q and r whose existence is asserted in the division algorithm ($n = qm + r$):

    (a) $11, -23$ $-23 = -3 \cdot 11 + 10$ $q = -3, r = 10$

    (b) $3, 42d + 5$ (where d is some integer) $42d + 5 = (42d + 3) + 2$
    $= 3(14d + 1) + 2$
    $q = 14d + 1$
    $r = 2$

4. Write the indicated note as a whole note, <u>choosing and notating an appropriate clef</u>.

    (a)   (b)   (c)

    B_2 $G^\sharp_5$ $E^\flat_4$

1

5. For the set $\mathbb{Z}$ and a fixed positive integer n, show that the relation $\equiv$ defined by $k \equiv \ell$ if and only if $n \mid (k - \ell)$ is an equivalence relation. Explain why there are exactly n equivalence classes.

(i) reflexive: $n \mid (k - k)$ since $k - k = 0 = 0 \cdot n$ so $k \equiv k$ for all k. Hence reflexive

(ii) symmetric: If $k \equiv \ell$ we have $k - \ell = mn$ for some $m \in \mathbb{Z}$. Multiplying by -1 we get $\ell - k = -mn$, so $n \mid (\ell - k)$ and $\ell \equiv k$. Hence symmetric

(iii) transitive: Assume $k \equiv \ell$ and $\ell \equiv p$ then $k - \ell = mn$ and $\ell - p = sn$. Adding, we get $(k - \ell) + (\ell - p) = mn + sn$
$$k - p = (m+s)n$$
which shows $n \mid (k - p)$ so $k \equiv p$. Hence Transitive.

So we have an equivalence relation

Given any $m \in \mathbb{Z}$, write $m = qn + r$ with $0 \le r < n$. (Euclidean algorithm) Then $m - r = qn$ so $m \equiv r$. So any integer is equivalent to one of the integers $0, 1, 2, \cdots, n-1$, of which there are n. These classes are distinct, since if $0 \le r < r' < n$, then $r' - r$ is too small to be divisible by n. So there are exactly n classes.

2
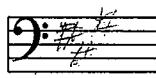
6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Lydian with tonic D

same as A major

(c) Phrygian with tonic C♯

same as A major

7. For each of the modes given below, name the chord type of the triad given by scale tones $\hat{2}$, $\hat{4}$, and $\hat{6}$.

(a) Ionian (major) *minor (D E A in C major)*

(b) Aeolian (minor) *diminished (B D F in A minor)*

(c) Locrian *major (C E G in B locrian)*

8. Extend the following melody with two measures having the same rhythm, employing the following transformations. Do not write in a key change.

(a) diatonic up two scale tones in the second measure

(b) chromatic up a major third (from the original) in the third measure

9. Give the total duration in beats of:

(a) a doubly-dotted quarter note in $\frac{2}{2}$ time. $\textit{♩} = \frac{1}{2} \text{ beat}$ $\textit{♩..} = \frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{4}\right)$
$$= \frac{1}{2}\left(\frac{7}{4}\right) = \boxed{\frac{7}{8} \text{ beat}}$$

(b) a half note in $\frac{9}{8}$ time (compound time signature). $\text{♪♩♪} = 1 \text{ beat}$
$$\textit{o} = \text{♪♩♪} \; ♪^{\curvearrowright} = \boxed{\frac{4}{3} \text{ beat}}$$

(c) an eighth note quintuplet in $\frac{4}{4}$ time.
$$\frac{1}{5} = \frac{1}{2^{n}\cdot 3} = \frac{1}{2^{n}\cdot r} \qquad \text{#}^{2} < 5 < 2^{3} \text{ so } r = 2, \text{ so } n = 1$$
$$\text{so } \text{♪♪}\overset{5}{\text{♪♪♪}} = \textit{o}^{\prime}, \text{ which has } \boxed{2 \text{ beats}}.$$

10. For the song *Mary Had A Little Lamb,* give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

Form: A B A C   (or A B A′ C)

rhythmic translation = m 2, 3, 4
melodic transposition = m 2, 3
   (diatonic and chromatic)