

# EXAM II

Math 109 / Music 109A, Spring 2018

Name \_\_\_\_\_ Id \_\_\_\_\_

Each problem is worth 10 points. Round off each decimal approximation to two digits to the right of the decimal.

- Express each of these musical intervals as an element of  $\mathbb{R}^+$  three ways:  
(1) as a power of 2, (2) as a radical or the reciprocal of a radical, and  
(3) by a decimal approximation.
  - down 46 cents
  - up a minor sixth
- Convert to the specified additive measurement the intervals given by the following ratios.
  - $19/16$ , convert to semitones
  - $\pi/6$ , convert to cents
- A string on a stringed instrument has length 50 cm. Indicate the positions of the single fret which will allow the string to play the note (a) a keyboard minor third above the original pitch, and (b) a ratio  $6/5$  with the original pitch.



7. Determine  $\phi(14)$  ( $\phi$  is the Euler phi function) by listing all the generating intervals in the 14-chromatic scale, represented as elements of  $\mathbb{Z}_{14}$ . Indicate which pairs of generating intervals are inverse to each other and for each pair draw the circle of intervals which is based on one element of the pair in the clockwise direction, the other element of the pair in the counterclockwise direction.

8. Determine whether or not each of the following pairs forms a monoid. If so, is it also also a group? Justify your answers.

(a)  $(\{-1, 0, 1\}, \cdot)$

(b)  $(\mathbb{Z}_m, +)$

9. Explain why the functions  $f(x) = 2^{x/12}$  and  $g(x) = 12 \log_2(x)$  are group homomorphisms, and why they are inverse to each other, thereby giving isomorphisms between the groups  $(\mathbb{R}, +)$  and  $(\mathbb{R}^+, \cdot)$ . Explain the connection of this with musical intervals.

10. Suppose a 12-tone row chart begins: G, A $\sharp$ , D $\flat$ , F, B, ... Write the upper left  $5 \times 5$  matrix of the resulting row chart. Then rewrite it replacing each note class with the element of  $\mathbb{Z}_{12}$  which measures its modular interval from G.