EXAM III
Math 109 / Music 109A, Spring 2010

Name ____________________________  Id __________________

Each problem is worth 10 points.

1. Define a (commutative) ring $R$ and show that $0 \cdot x = 0$, for any $x \in R$.
   Use this to show that if $0 = 1$ in $R$, then $R = \{0\}$.

2. Determine whether these subset of $\mathbb{Z}$ are ideals. If so, express them in the form $n\mathbb{Z}$, where $n$ is a positive integer:
   (a) the odd integers
   (b) $20\mathbb{Z} + 14\mathbb{Z}$
   (c) $(-17)\mathbb{Z}$
   (d) the negative integers
3. Prove that there are infinitely many prime numbers.

4. Using unique factorization in $\mathbb{Q}^+$, explain why any rational interval can be written uniquely as a composition of prime intervals and their opposites.

5. On the staff system below, write the keyboard’s best approximation for each harmonic up through 11 for the indicated note. For harmonics 5, 7 and 11, indicate how sharp or flat (to the nearest cent) the keyboard’s approximation is.
6. Find the value $\gamma$ for which the pitch associated to the periodic function $h(t) = d \sin(\gamma t + \beta)$, where $t$ is time in seconds, is $A_4^\flat$.

7. Find the period, frequency, amplitude, and phase shift for the function 

$$g(t) = \sqrt{3} \sin(440\pi t) + \cos(440\pi t)$$

and express it in the form $d \sin(\alpha t + \beta)$, giving a decimal approximation for $\beta$. 
8. We established that the square wave, defined on \([0, 2\pi]\) by

\[
s(t) = \begin{cases} 
1, & \text{for } 0 \leq t < \pi \\
-1, & \text{for } \pi \leq t < 2\pi 
\end{cases}
\]

has Fourier series

\[
s(t) = \sin t + \frac{1}{3} \sin(3t) + \frac{1}{5} \sin(5t) + \cdots
\]

Draw the graph of \(s(t)\). Give the values of the Fourier coefficients \(C, A_k, B_k\) for \(k \in \mathbb{Z}^+\), and indicate the amplitude and phase shift of each harmonic. By evaluating an integral using areas, verify that the values of \(C\) and \(B_1\) are what you have determined them to be.
9. A certain vowel sound has a formant which amplifies frequencies within 300 Hz of 2500 Hz. A baritone sings the vowel at C\textsubscript{3}. Which harmonics are amplified?

10. (a) Express the just major third as a rational number with prime factorization and in cents, rounding off the latter at 2 digits to the right of the decimal.

(b) Do the same for the comma of Didymus, which is the “difference” between the greater and lesser whole tones.