EXAM III

Math 109 / Music 109A, Spring 2013

Name ______________________________ Id ____________________

Each problem is worth 10 points.

1. Define a (commutative) ring R and show that $0 \cdot x = 0$, for any $x \in R$. Use this to show that $(-1) \cdot x = -x$, for any $x \in R$.

2. Determine whether these subsets of $\mathbb{Z}$ are ideals. If so, express them in the form $n\mathbb{Z}$, where n is a positive integer. Justify your answers.

   (a) the positive integers

   (b) $100\mathbb{Z} + 30\mathbb{Z}$

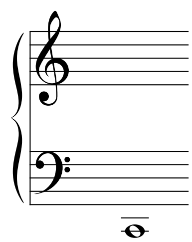
3. Prove that there are infinitely many prime numbers.

4. List the units of the ring $\mathbb{Z}_{12}$ and explain the implication this has for keyboard intervals.

5. Give the prime factorizations of these integers, writing the primes in ascending order, as in $2^3 \cdot 3^1 \cdot 7^2$.

   (a) 75 (b) 64 (c) 242 (d) 52 (e) 14×10^{23}

6. On the staff system below, write the keyboard's best approximation for harmonics 1 through 11 for the indicated note. For the 11th harmonic, indicate how sharp or flat (to the nearest cent) the keyboard's approximation is.



7. Find the value γ for which the pitch associated to the periodic function $h(t) = d\sin(\gamma t + \beta)$, where t is time in seconds, is $\mathrm{E}^\flat_4$.

8. Find the period, frequency, amplitude, and phase shift for the function

$$g(t) = 3\sin(880\pi t) + 4\cos(880\pi t)$$

and express it in the form $d\sin(\alpha t+\beta)$, giving a decimal approximation for β.

9. A certain vowel sound has a formant which amplifies frequencies within 350 Hz of 2900 Hz. An alto singer sings the vowel at A_3. Which harmonics are amplified?

10. We established that the square wave, defined on $[0, 2\pi)$ by

$$s(t) = \begin{cases} 1, & \text{for } 0 \leq t < \pi \\ -1, & \text{for } \pi \leq t < 2\pi \end{cases}$$

has Fourier series

$$s(t) = \frac{4}{\pi}\left(\sin t + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots\right)$$

Draw the graph of $s(t)$. Give the values of the Fourier coefficients C, A_k, B_k for $k \in \mathbb{Z}^+$, and indicate the amplitude and phase shift of each harmonic. By evaluating an integral using areas, verify that the value of B_1 is what you have determined it to be.