EXAM III

Math 109 / Music 109A, Spring 2014

Name ______________________ Id ______________

Each problem is worth 10 points.

1. Define a (commutative) ring R and show that $0 \cdot x = 0$, for any $x \in R$. Use this to show that if $0 = 1$ in R, then $R = \{0\}$.

2. Determine whether these subset of $\mathbb{Z}$ are ideals. If so, express them in the form $n\mathbb{Z}$, where n is a positive integer:

   (a) the odd integers

   (b) $20\mathbb{Z} + 14\mathbb{Z}$

   (c) $(-17)\mathbb{Z}$

   (d) the negative integers

3. Prove that there are infinitely many prime numbers.

4. Using unique factorization in $\mathbb{Q}^+$, explain why any rational interval can be written uniquely as a composition of prime intervals and their opposites.

5. On the staff system below, write the keyboard's best approximation for each harmonic up through 11 for the indicated note. For harmonics 5 and 11, indicate how sharp or flat (to the nearest cent) the keyboard's approximation is.

6. Find the value γ for which the pitch associated to the periodic function $h(t) = d\sin(\gamma t + \beta)$, where t is time in seconds, is $\mathrm{A}_4^\flat$.

7. Find the period, frequency, amplitude, and phase shift for the function

$$g(t) = \sqrt{3}\, sin(440\pi t) + \cos(440\pi t)$$

and express it in the form $d\sin(\alpha t + \beta)$, giving a decimal approximation for β.

8. List the elements of the group of units $\mathbb{Z}_9^*$? Is this group cyclic? If so find all generators.

9. A certain vowel sound has a formant which amplifies frequencies within 300 Hz of 2500 Hz. A baritone sings the vowel at C_3. Which harmonics are amplified?

10. Recall the general form of the Fourier series:

$$f(t) = C + \sum [A_k \sin(kt) + B_k \cos(kt)] \,.$$

We established that the square wave, defined on $[0, 2\pi)$ by

$$s(t) = \begin{cases} 1, & \text{for } 0 \le t < \pi \\ -1, & \text{for } \pi \le t < 2\pi \end{cases}$$

and extended by periodicity, has Fourier series

$$s(t) = \frac{4}{\pi}\left(\sin t + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots\right)$$

Draw the graph of $s(t)$. From the above sum identify the values of the Fourier coefficients C, A_k, B_k for $k \in \mathbb{Z}^+$, and indicate the amplitude and phase shift of each harmonic. By evaluating an integral using areas, verify that the value of A_1 is what you have determined them to be.