EXAM III

Math 109 / Music 109A, Fall 2022

Name Solutions Id ______________

Each problem is worth 10 points. Round off each decimal approximation to two digits to the right of the decimal.

1. Which of the following sets, together with with given operation, form a monoid, and which are also a group? Justify your answers.

   (a) $\mathbb{R}^+ , \cdot$ (b) $\mathbb{Z} , \cdot$ (c) $\{0, 1, 2\}, +$

   (a) closed with respect to $\cdot$, 1 is identity element, $x \in \mathbb{R}^+$ has inverse $\frac{1}{x}$. monoid & group.

   (b) closed with respect to $\cdot$, 1 is identity, so monoid. only $1, -1$ have inverses in $\mathbb{Z}$, so not a group

   (c) not closed with respect to $+$, as $1+2 \notin \{0,1,2\}$. neither.

2. Consider the twelve-tone row:

   Write the first two rows and the first column of the row chart having the above as its original row.

   B E F# A A# F C G D# G# C# D
   F# B C# E F C G D A# D# G# A
   E
   C#
   C
   F
   A#
   D#
   G
   D
   A
   G#

3. Create n-tone row charts for the following choices of n and the given sequences of original rows in $\mathbb{Z}_n$:

(a) $n = 4$; ([0], [3], [1], [2])

(b) $n = 5$; ([0], [2], [4], [1], [3])

(a)

[0]	[3]	[1]	[2]
[1]	[0]	[2]	[3]
[3]	[2]	[0]	[1]
[2]	[1]	[3]	[0]

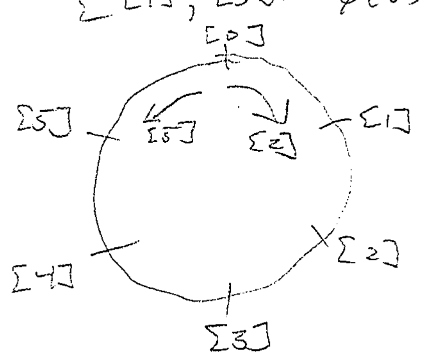
(b)

[0]	[2]	[4]	[1]	[3]
[3]	[0]	[2]	[4]	[1]
[1]	[3]	[0]	[2]	[4]
[4]	[1]	[3]	[0]	[2]
[2]	[4]	[1]	[3]	[0]

4. For each of these choices of n, determine $\phi(n)$ (ϕ is the Euler Phi function) by listing all the generating intervals in the n-chromatic scale. Indicate which pairs of generating intervals are inverse to each other and for each pair draw the circle of intervals which is based on one element of the pair in the clockwise direction, the other element of the pair in the counterclockwise direction.

(a) n=6

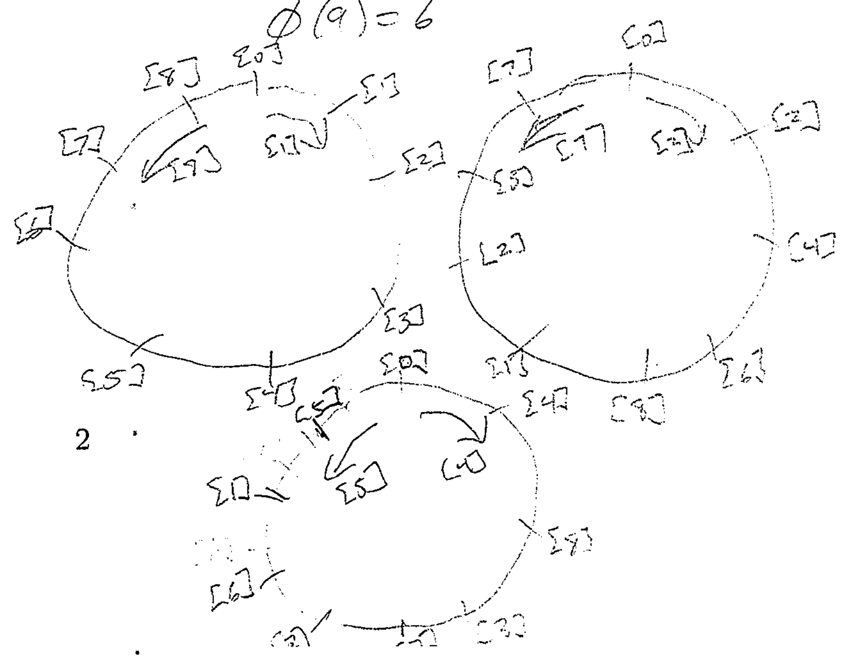
generators $\{[1], [5]\}$ $\phi(6) = 2$



(b) n=9

generators $\{[1], [2], [4], [5], [7], [8]\}$

$\phi(9) = 6$



5. Prove that if $y = f(t)$ has period P, 1 then $y = cf(t)$ has period P, but
2 $y = f(t/c)$ $(c \neq 0)$ has period cP.

2 Let $g(t) = f\left(\frac{t}{c}\right)$

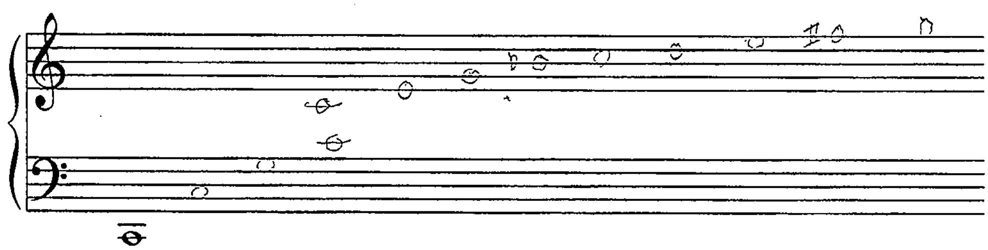
$$g(t + cP) = f\left(\frac{t + cP}{c}\right) = f\left(\frac{t}{c} + P\right) = f\left(\frac{t}{c}\right) = g(t)$$

So g has period cP

1 Let $h(t) = c f(t)$. $h(t+P) = c f(t+P) = c f(t) = h(t)$

6. The following is in the major mode. In each of measures 2 and 3, write in a <u>seventh</u> chord of whole notes that makes the passage resolve around the circle of fifths (going counterclockwise toward I). Write each chord in root position, meaning the root is the lowest note, with two notes on each clef, as if it were SATB. Label each chord, including the first one, by root note class below the chord and by Roman numeral below the chord, each with appropriate suffix.

7. On the staff system below, write the keyboard's best approximation for harmonics 1 through 12 for the indicated note. For the 5th, 9th, and 11th harmonics, indicate how sharp or flat (to the nearest cent) the keyboard's approximation is. For which harmonics is the keyboard note exact?



1 2 3 4 5 6 7 8 9 10 11 12

5: $1200 \log_2 5 = 2786.31$ 14 cents sharp

9 - $1200 \log_2 9 = 3803.91$ 4 cents flat

11: $1200 \log_2 11 = 4151.32$ 49 cents sharp

2, 4, 8 are exact, being powers of 2.

8. Find the value γ for which the pitch associated to the periodic function $h(t) = d\sin(\gamma t + \beta)$, where t is time in seconds, is $E^\flat_5$. (Assume A_4 is tunes to 440 Hz.)

$E_2 = 440 \cdot 2^{6/12} \approx 622.254 = F$

$\gamma = 2\pi F \approx 1244.51\pi \approx 3909.74$

9. Find the period, frequency, amplitude, and phase shift for the function

$$g(t) = 3\sin(440\pi t) + 4\cos(440\pi t)$$

and express it in the form $d\sin(\alpha t+\beta)$, giving a decimal approximation for β.

$440\pi = 2\pi F$ so $F = 220$ $(= A_3)$

period $P = \frac{1}{F} = \frac{1}{220}$

$d = \sqrt{3^2+4^2} = \sqrt{25} = 5$ = amplitude

$g(t) = 5\left(\frac{3}{5}\sin(440\pi t) + \frac{4}{5}\cos(440\pi t)\right)$

$= 5\sin(440\pi t + \beta)$

where $\beta = \arcsin\frac{4}{5} \approx 0.93$ phase shift

10. A certain vowel sound has a formant which amplifies frequencies within 350 Hz of 2900 Hz. An alto singer sings the vowel at A_3. Which harmonics are amplified in this formant?

$A_3 = 220$

$2550 \leq 220n \leq 3250$

$\frac{2550}{220} \leq n \leq \frac{3250}{220}$

$11.59 \leq n \leq 14.77$

$n = 12, 13, 14$