FINAL EXAM

Math 109 / Music 109A, Spring 2015

Name _____ Id _____

Each problem is worth 10 points. Round off each decimal approximation to two digits to the right of the decimal.

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1. Give the (total) duration in beats of:

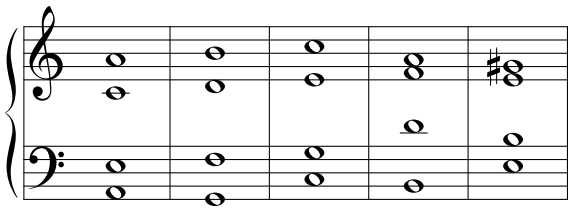
   (a) a sixteenth note in $\frac{12}{8}$ time (compound time signature).

   (b) a dotted eighth note in $\frac{2}{2}$ time.

   (c) a quarter note 5-tuplet in $\frac{4}{4}$ time.

2. For the set $\mathbb{Z}$ and a fixed positive integer m, define the equivalence relation whose set of equivalence classes is $\mathbb{Z}_m$. Show that it is in fact an equivalence relation and explain why there are exactly m equivalence classes. For $m = 12$ explain how this relates to keyboard intervals.

3. Identify each chord in this minor mode (Aeolian) passage. Above the staff label each chord by root note class with suffix (e.g., $E^{\flat 7}$). Below the staff, label each chord by root scale tone (e.g. $\flat III^7$).



4. Convert to semitones the musical intervals given by the following ratios, indicating whether the interval is upward or downward.

   (a) 0.4

   (b) $\frac{\pi}{4}$

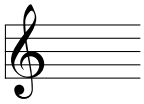
   Express as a ratio the following (upward) musical intervals.

   (c) 86 cents

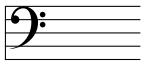
   (d) the just major third

5. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef. Place the sharps and flats in their proper positions.

(a) Locrian with tonic D



(b) Myxolydian with tonic $B^\flat$



6. Suppose a 12-tone row chart begins: A, $E^\flat$, $G^\flat$, F, $C^\sharp$, ... Write the upper left 5×5 matrix of the resulting row chart. Then rewrite it replacing each note class with the element of $\mathbb{Z}_{12}$ which measures its modular interval from A.

7. If the keyboard divided the octave into 10 rather than 12 equal intervals, what would be the generating intervals? Quote the relevant facts from modular arithmetic that justify your answer. Draw the circles that exhibit inverse pairs of these generating intervals.

8. (a) Find the period, frequency, amplitude, and phase shift for the function

$$g(t) = 3\sin(440\pi t) + 4\cos(440\pi t)$$

and express it in the form $d\sin(\alpha t+\beta)$, giving a decimal approximation for β. Identify the closest keyboard note to the pitch represented by this function, and the error (if any) in cents.

(b) Find the period, frequency, amplitude, and phase shift for the function

$$h(t) = 2\sqrt{2}\sin\left(660\pi t + \frac{\pi}{4}\right)$$

and express it in the form $A\sin\alpha t + B\cos\alpha t$. Identify the closest keyboard note to the pitch represented by this function, and the error (if any) in cents.

9. We established that the square wave, defined on $[0, 2\pi)$ by

$$s(t) = \begin{cases} 1, & \text{for } 0 \le t < \pi \\ -1, & \text{for } \pi \le t < 2\pi \end{cases}$$

has Fourier series

$$s(t) = \frac{4}{\pi}\left(\sin t + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots\right)$$

Draw the graph of $s(t)$. Give the values of the Fourier coefficients C, A_k, B_k for $k \in \mathbb{Z}^+$, and indicate the amplitude and phase shift of each harmonic which is actually present. By evaluating an integral using areas, verify that the value of B_1 is what you have determined it to be. The only needed fact from calculus is

$$\int_0^\pi \sin x \, dx = 2 \ .$$

10. (a) Explain the comma of Pythagoras. How does it arise? Express it as a rational number and in cents.

(b) For each scale tone $\hat{1}$ to $\hat{8}$, write the fraction that expresses the ratio of that scale tone to $\hat{1}$ in the just intonation diatonic scale. Factor each fraction into primes.

$\hat{1}$

$\hat{2}$

$\hat{3}$

$\hat{4}$

$\hat{5}$

$\hat{6}$

$\hat{7}$

$\hat{8}$

Express the interval from $\hat{2}$ to $\hat{6}$ as a fraction and in cents. Is this the just fifth?

HAVE A GOOD SUMMER!