Homework 5

Math 109 / Music 109A, Spring 2014

Due Monday, March 31.

1. Prove that in any (commutative) ring R we have the following two equalities, for any $x \in R$:

   (a) $0 \cdot x = 0$

   (b) $(-1) \cdot x = -x$

2. Express each of these ideals in $\mathbb{Z}$ in the form $n\mathbb{Z}$, where n is a positive integer:

   (a) the even integers

   (b) $(-20)\mathbb{Z}$

   (c) $18\mathbb{Z} + 44\mathbb{Z}$

   (d) $13\mathbb{Z} + 35\mathbb{Z}$

3. Find and list all of the ideals in $\mathbb{Z}_{14}$. Which of these are principal ideals?

4. Prove that there are infinitely many prime numbers. (Hint: If $p_1, \ldots, p_n$ were a complete list of primes, consider a prime factor of $p_1 \cdots p_n + 1$.)

5. Prove that if p is prime and $n \in \mathbb{Z}$, then either $p \mid n$ or $\gcd(p, n) = 1$.

6. Give the prime factorizations of these integers, writing the primes in ascending order, as in $2^3 \cdot 3 \cdot 7^2$.

   (a) 110 (b) 792 (c) 343 (d) 3422 (e) 15×10^{23}

7. List the primes larger than 13 but less than 50, and for each, determine how closely its musical interval is approximated by the keyboard, calculating the error in cents. Which primes have good keyboard approximations, where "good" means within 20 cents?

8. Call a musical interval a *prime interval* if its interval ratio is a prime integer; call it a *rational interval* if its interval ratio is a rational number. Show that all rational intervals can be written as compositions of prime intervals and their opposites.

9. What is the group of units $\mathbb{Z}_7{}^*$? Is this group cyclic? If so find all generators.

10. Compose a brief melodic passage using one of the m on n techniques discussed at the end of this Chapter 8. Explain why it works. Just a very few bars will suffice.