Homework 5

Math 109 / Music 109A, Spring 2022

Due Monday, November 14.

1. Which of the following sets, together with with given operation, form a monoid, and which are also a group? Justify your answers.

   (a) $\mathbb{R}$, $\cdot$ (b) $\mathbb{Z}$, $+$ (c) $\{1, -1\}$, $\cdot$ (d) $\{-1, 0, 1\}$, $+$

2. Perform each on these arithmetic operations in the specified $\mathbb{Z}_n$, expressing your answer as $[r]$ where $0 \le r < n$.

   (a) $[7]+[11]+[9]$ in $\mathbb{Z}_{12}$

   (b) $[17]-[25]+[37]$ in $\mathbb{Z}_5$

   (c) $[13]-[4]+[6]$ in $\mathbb{Z}_2$

3. Express the following compositions of modular 12-chromatic intervals as r semitones with $0 \le r < 12$. Interpret all these compositions as operations in the additive group $\mathbb{Z}_{12}$. (Intervals are upward unless otherwise noted.)

   (a) the composition of 15 and 19 semitones

   (b) two minor sevenths and a major third

   (c) six fourths

   (d) up five major thirds, down three tritones

4. For each of these choices of n, determine $\phi(n)$ (ϕ is the Euler Phi function) by listing all the generating intervals in the n-chromatic scale. Indicate which pairs of generating intervals are inverse to each other and for each pair draw the circle of intervals which is based on one element of the pair in the clockwise direction, the other element of the pair in the counterclockwise direction.

   (a) $n=4$ (b) $n=5$ (c) $n=10$ (d) $n=14$

5. Create a twelve-tone row chart having this sequence as its original row:
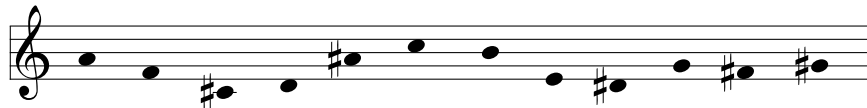
6. Translate the row chart from the previous problem to a 12×12 matrix with entries in $\mathbb{Z}_{12}$ with $[0]$ as the upper left entry.

7. Create n-tone row charts for the following choices of n and the given sequences of original rows in $\mathbb{Z}_n$:

   (a) $n = 3$; $([0], [2], [1])$

   (b) $n = 4$; $([0], [2], [3], [1])$

   (c) $n = 6$; $([0], [2], [3], [1], [4], [5])$

   (d) $n = 7$; $([0], [3], [1], [5], [2], [6], [4])$

8. Each integer n determines a musical interval as a ratio. On the staff system below, write the keyboard's best approximation for the note having ratio n to the given note, for $n \in \{2, 3, 4, 5, 6, 7, 8\}$. For each, indicate how sharp or flat (to the nearest cent) the keyboard's approximation is.

9. The following is in the major mode. In each of measures 2 and 3, write in a <u>seventh</u> chord of whole notes that makes the passage resolve around the circle of fifths (going counterclockwise toward I). Write each chord in root position, meaning the root is the lowest note, with two notes on each clef, as if it were SATB. Label each chord, including the first one, by root note class and by Roman numeral, each with suffix. should be the those indicated under the staff.

10. Analyze the basic harmony in the first 16 measures of *Maple Leaf Rag.* Each measure will have at most two chords. Label the chords with appropriate suffix by root note class (e.g., IIm) and chord type (e.g., G^7). (Note: In a few places the chords are incomplete.) The music can be downloaded as a pdf file from the website.