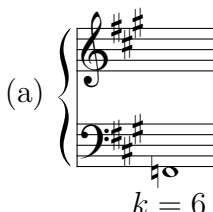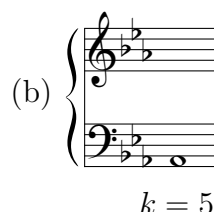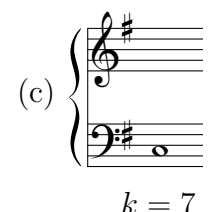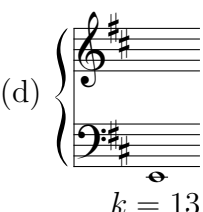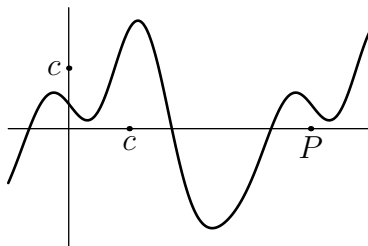# Homework 6

Math 109 / Music 109A, Fall 2022

Due Monday, December 5.

1. For each given note N and integer k: label N by letter and subscript (e.g., $A_4^\flat$); write on the staff the (12-chromatic) note M which best approximates the kth harmonic of N. Label M by letter and subscript.

2. In the sequence of keyboard approximations of the integer ratios 1 through 12, find all sets of four adjacent notes which can be identified as one of the chords listed in Chapter 3. Find the other chords which appear within the entire sequence.

3. Show that if $y = f(t)$ has period P, then $y = f(t/c)$ $(c \neq 0)$ has period cP.

4. Suppose the function $y = f(t)$ is the periodic function of period P corresponding to a musical tone, and suppose the graph of $y = f(t)$ is:

For each of the functions below, sketch its graph and explain how its associated tone compares that of $f(t)$.

(a) $y = \frac{1}{2}f(t)$

(b) $y = f(2t)$

(c) $y = f(t) + c$

(d) $y = f(t + c)$

5. Assuming A_4 is tuned to 440 Hz, find the value α for which the pitch associated to the periodic function $y = \sin(\alpha t)$, where t is time in seconds, is:

(a) middle C

(b) $A_2^\flat$

(c) $D_6^\sharp$

6. Find the period, frequency, amplitude, and phase shift for these functions, and express each in the form $A\sin(\alpha t) + B\cos(\alpha t)$:

(a) $f(t) = 5\sin(30\pi t + \frac{\pi}{4})$

(b) $g(t) = \sqrt{2}\sin(800t + \pi)$

7. Find the period, frequency, amplitude, and phase shift for these functions, and express each in the form $d\sin(\alpha t + \beta)$:

(a) $f(t) = 4\sin(300t) + 5\cos(300t)$

(b) $h(t) = -\sin(1500\pi t) + 3\cos(1500\pi t)$

8. A certain soprano's *ee* vowel has a formant centered at 2900 Hz. What pitch, in Hz, should she sing in order for the fifth harmonic to be maximally amplified by this formant?

9. Two instruments play the keyboard pitches A$_2$ and E$_3$, making the interval of a keyboard fifth. Suppose they are the same kind of instrument, and that the instrument has a formant centered at 3000 Hz. Suppose the formant amplifies pitches within 400 Hz of its center. Identify the harmonics produced by each instrument which will be amplified by the formant, and give their frequencies. How many pairs of these frequencies are almost aligned? Could this "near alignment" be perfected by slightly adjusting the interval?

10. The Fourier series for the function $g(t)$ has the form

$$g(t) = 0.2\sin(880\pi t) + 0.1\sin(1760\pi t) + 2\sin(2640\pi t)$$

This means that all amplitudes d_k with $k \geq 4$ are zero, and all phase shifts β_k are zero. What is the fundamental pitch of this tone? What pitch, other than the fundamental, is likely to be heard when the tone is sounded?