

## A COMPARISON THEOREM FOR NEUMANN HEAT KERNELS IN THE UNIT BALL IN $\mathbb{R}^n$

ABSTRACT. In this talk, I will discuss a comparison theorem concerning the Neumann heat kernel for the unit ball  $U$  in  $\mathbb{R}^n$ . Viewing the heat kernel  $p(t, x, \cdot)$  as the solution to the heat problem

$$\begin{cases} \partial_t u = \Delta u & \text{in } U \\ \partial_\nu u = 0 & \text{on } \partial U \end{cases}$$

with a Dirac mass  $\delta_x$  as the initial data, the theorem reads as follows: *polarizing the data in towards the center decreases the heat kernel pointwise within the cap cut out by the hyperplane of polarization*. The proof, due to Pascu and Gageonea, involves a construction from probability theory, called *mirror coupled reflecting Brownian motion*, which I will describe. If time permits, I will show how Pascu and Gageonea's result yields the following theorem, originally conjectured by Rick Laugesen and Carlo Morpurgo:  *$p(t, x, x)$  is a radially increasing function*.