## ABSTRACTS OF LECTURES

Wednesday, May 11, 2:15 PM - 3:05 PM

## THE GROWTH OF UNIVALENT FUNCTIONS WITH AN INITIAL GAP

Walter K. Hayman

Imperial College London
(Joint work with Dov Aharonov and Christian Pommerenke)

We consider the class $S_{p}$ of those functions

$$
f(z)=z+a_{p+1} z^{p+1}+\ldots
$$

univalent in the unit disk $\Delta$. We show that if $f \in S_{p}$, where $p$ is large, then

$$
\alpha(f)=\lim _{n \rightarrow \infty} \frac{\left|a_{n}\right|}{n} \leq \alpha(p)=\frac{2(\log p \log \log p)^{2}}{p^{4}}
$$

We also obtain upper bounds for $\left|a_{n}\right|$ when $f \in S_{p}$.
In the opposite direction, examples are constructed of functions $f \in S_{p}$ for which $\alpha(f) \geq$ $C_{0} p^{-4}$, where $C_{0}$ is a positive absolute constant.

## Wednesday, May 11, 3:20 PM - 3:45 Pm

## SCHWARZIAN DERIVATIVES AND UNIVALENCE OF HARMONIC MAPPINGS

## Peter Duren

University of Michigan, Ann Arbor
(Joint work with Martin Chuaqui and Brad Osgood)

In collaboration with Martin Chuaqui and Brad Osgood, the speaker recently extended the classical notion of Schwarzian derivative of an analytic function to general complexvalued harmonic functions $f(z)=h(z)+\overline{g(z)}$, where $h$ and $g$ are analytic. With respect to the conformal parameter $\lambda=\left|h^{\prime}\right|+\left|g^{\prime}\right|$ of the associated minimal surface, the definition is

$$
\mathcal{S} f=2\left\{(\log \lambda)_{z z}-(\log \lambda)_{z}^{2}\right\}
$$

Nehari's well-known general criterion (1954) for univalence of an analytic function in the unit disk is now generalized to the Weierstrass-Enneper lift of a harmonic mapping $f$ to its minimal surface. The criterion involves $\mathcal{S} f$ and the Gauss curvature of the surface, which can also be computed in terms of $\lambda$.

# Wednesday, May 11, 4:40 PM - 5:05 PM <br> ON THE PRODUCT OF FUNCTIONS IN BMO AND H ${ }^{1}$ 

Tadeusz Iwaniec<br>Syracuse University

(Joint work with Aline Bonami, Peter Jones, and Michel Zinsmeister)

The point-wise product $\mathfrak{b} \cdot \mathfrak{h}$ of functions $\mathfrak{b} \in \boldsymbol{B M O}\left(\mathbb{R}^{n}\right)$ and $\mathfrak{h} \in \boldsymbol{H}^{1}\left(\mathbb{R}^{n}\right)$ need not be locally integrable. However, in view of the duality between $B M O$ and $\boldsymbol{H}^{1}$, we are able to give a meaning to $\mathfrak{b} \cdot \mathfrak{h}$ as a Schwartz distribution, denoted by $\mathfrak{b} \times \mathfrak{h} \in \mathcal{D}^{\prime}\left(\mathbb{R}^{n}\right)$. The central question is concerned with the regularity of $\mathfrak{b} \times \mathfrak{h} \in \mathcal{D}^{\prime}\left(\mathbb{R}^{n}\right)$. We prove a decomposition:

$$
\mathfrak{b} \times \mathfrak{h}=\alpha+\beta,
$$

where $\alpha$ is a function in $\boldsymbol{L}^{1}\left(\mathbb{R}^{n}\right)$ while $\beta$ is a distribution in a Hardy-Orlicz space. Precisely this means that its maximal function $\mathcal{M} \beta$ satisfies

$$
\int_{\mathbb{R}^{n}} \frac{\mathcal{M} \beta}{\log (e+\mathcal{M} \beta)} d \mu<\infty, \quad \text { where } \quad d \mu=\frac{d x}{\log (e+|x|)}
$$

The Jacobian determinants and more general div-curl products come to a play as atoms.

Thursday, May 12, 8:15 am - 9:45 am

## p-LAPLACE WORKSHOP

Yuanji Cheng ${ }^{1}$ and Juan Manfredi ${ }^{2}$
${ }^{1}$ Malmö University
${ }^{2}$ University of Pittsburgh

We will present an introduction to the $p$-Laplace equation for graduate students according to the following outline:
(1) The case $1<p<\infty$; various notions of weak solutions and regularity.
(2) The limit cases $p=\infty$ and $p=1$.
(3) The two-dimensional case; connections with quasiregular mappings.
(4) $p$-Harmonic measure.
(5) The $p$-Laplacian relative to a set of Hörmander vector fields.

Thursday, May 12, 10:00 Am - 10:50 Am

# DIFFERENTIAL POLYNOMIALS WITH REAL ZEROS 

Alexander Eremenko
Purdue University

This is a survey of recent results on the following question: Let $f$ be a real meromorphic function, and $P(f)$ a real differential polynomial of $f$. What can be said about $f$ if $P(f)$ has only real zeros? The main result is the old conjecture of Wiman recently proved by Bergweiler, Langley and the speaker: if $f$ is a real entire function, and all zeros of $f f^{\prime \prime}$ are real, then $f$ is a limit of polynomials with real zeros. Various generalizations of this theorem and open questions will be discussed.

Thursday, May 12, 11:05 AM - 11:30 Am

# CONTINUOUS SYMMETRIZATION VIA POLARIZATION 

Alexander Solynin<br>Texas Tech University and Steklov Institute of Mathematics<br>(Joint work with Friedemann Brock)

We will discuss a one-parameter family of transformations which transforms in a continuous way sets and functions into their $(k, n)$-Steiner symmetrizations. Our construction consists of two stages. On the first stage we employ a continuous symmetrization introduced in our paper in 1990 to transform sets and functions into their one-dimensional Steiner symmetrization. On this step some of our proofs depend on polarization.

On the second stage we use Blaschke-Sarvas approximation theorem, which says that every $k$-dimensional Steiner symmetrization with $2 \leq k \leq n$ can be approximated by a sequence of $(k-1)$-dimensional Steiner symmetrizations. Using this result we give an inductive definition of a continuous ( $k, n$ )-Steiner symmetrization for any $2 \leq k \leq n$. This transformation gives the desired continuous path along which all basic characteristics of sets and functions change monotonically. The latter leads to continuous versions of several convolution type inequalities and Dirichlet's type inequalities and to continuous versions of comparison theorems for solutions of some elliptic and parabolic equations.

Thursday, May 12, 11:45 am - 12:10 Pm

## GROWTH OF SOLUTIONS TO THE MINIMAL SURFACE EQUATION

## Allen Weitsman

Purdue University
Let $L$ denote the minimal surface operator. Consider the problem

$$
\begin{cases}L u=0 & \text { in } D, \\ u=0 & \text { on } \partial D,\end{cases}
$$

with $u>0$ in $D$. By the maximum principle, $D$ must be unbounded. I will discuss problems involving upper and lower bounds on the growth of solutions.

## Thursday, May 12, 3:05 PM - 3:55 PM

# THE BILIPSCHITZ INVARIANCE OF LIPSCHITZ HARMONIC CAPACITY FOR COMPACT SUBSETS OF $\mathbb{R}^{d}$. 

## John Garnett

University of California, Los Angeles
(Joint work with Laura Prat)

We show that if $E$ is a Cantor set in $\mathbb{R}^{d}$ formed by intersecting a decreasing family of sets $E_{n}$ where $E_{n}$ consists of $2^{\text {nd }}$ cubes in $\mathbb{R}^{d}$ of side $\sigma_{m}$ and where the components of $E_{n+1}$ are the corners of the components of $E_{n}$, if $T$ is a bilipschitz self map of $\mathbb{R}^{d}$, and if $\kappa$ is the Lipschitz harmonic capacity, then

$$
\kappa(T(E)) \leq C_{T} \kappa(E) .
$$

The proof uses the theorem of Mateo and Tolsa that

$$
\kappa(E) \sim\left(\sum \frac{2^{n} d}{\sigma_{n}^{d-1}}\right)^{\frac{1}{2}}
$$

and other things.

Thursday, May 12, 4:10 PM $-4: 35$ PM

ON p-HARMONIC MEASURE<br>Jang-Mei Wu<br>University of Illinois, Urbana-Champaign

Unlike the case $p=2$, known results on $p$-harmonic measure for $p \neq 2$ are sporadic; structure of sets having zero $p$-harmonic measure is largely unknown. We first give a survey, then discuss a recent example on non-subaddivity due to Llorente, Manfredi and the speaker. In fact, there are finitely many sets $E_{1}, E_{2}, \ldots, E_{\kappa}$ on $\mathbb{R}^{1}$, such that each has zero $p$-harmonic measure with respect to the half plane $\mathbb{R}_{+}^{2}$, on the other hand, their union is $\mathbb{R}^{1}$.

The construction is motivated by work of Al Baernstein, his students, students of his students, and many others.

# Friday, May 13, 10:00 AM - 10:50 AM <br> p-HARMONIC FUNCTIONS: THEY ARE NOT JUST FOR BREAKFAST ANYMORE 

John Lewis<br>University of Kentucky

In this talk I plan to first drudge up some old memories concerning how Albert Baernstein II and the Baernstein $*$-function influenced my departure from the land of Complex Analysis for the realm of Partial Differential Equations. Second I will discuss recent work with coauthors on $p$-harmonic functions, which hopefully will reestablish my credentials to be considered at least partially a complex analyst.

Friday, May 13, 11:05 AM - 11:30 AM

## SOME TOPICS IN MULTIPARAMETER FOURIER ANALYSIS

## Jill Pipher

Brown University

I'll start with a quick introduction to the main issues in this field, survey some recent results, and sketch some directions for further research. All in 25 minutes!

Friday, May 13, 11:45 AM - 12:10 PM

AN ANALOG OF HADAMARD'S DETERMINANT INEQUALITY FOR PERMANENTS<br>\section*{Michael Loss}<br>Georgia Tech University

(Joint work with Eric Carlen and Elliott Lieb)
Let $F$ be an $N$ by $N$ matrix and denote by $\vec{f}_{j}$ the $j$-th column of the matrix $F$ and $\left|\vec{f}_{j}\right|$ its Euclidean length. Hadamard's inequality states that

$$
|\operatorname{det} F| \leq \prod_{j=1}^{N}\left|\overrightarrow{f_{j}}\right| .
$$

Here we shall prove that

$$
|\operatorname{perm}(F)| \leq \frac{N!}{N^{N / 2}} \prod_{j=1}^{N}\left|\vec{f}_{j}\right|,
$$

with equality when $F$ is a constant matrix.

## Friday, May 13, 3:05 PM - 3:55 PM

## MARTINGALES AND THE BEURLING-AHLFORS OPERATOR, AN OVERVIEW

Rodrigo Bañuelos

Purdue University

This will be an overview talk discussing Burkholder's sharp inequalities for martingales and their use to derive information on the $L^{p}$-norms of Riesz transforms and related singular integrals. In particular, we will describe progress made in recent years via martingales on the celebrated conjecture of Tadeusz Iwaniec concerning the $L^{p}$-norms of the Beurling-Ahlfors operator.

Friday, May 13, 4:10 PM - 4:35 PM

# SOME INEQUALITIES FOR CONTINUOUS MARTINGALES 

## Burgess Davis

Purdue University

If $X(t), t \geq 0$, is a continuous martingale we let $M=\sup |X(t)|$ and let $Q$ be the quadratic variation of $X$. Burkholder has recently revisited and sharpened some of his and Gundy's celebrated moment inequalities relating $M$ and $Q$. I will talk about his work and a follow up paper by Jiyeon Suh and myself. For the continuous martingale associated with a harmonic function in the disc, $M$ and $Q$ are called the Brownian maximal and area functions, and the relevance of the martingale results in this setting will be described.

# Saturday, May 14, 11:05 am - 11:30 am <br> IMPROVED PAINLEVÉ REMOVABILITY FOR K-QUASIREGULAR MAPPINGS 

Kari Astala<br>University of Helsinki

(Joint work with A. Clop, J. Mateu, J. Orobitg and I. Uriarte-Tuero)

Painlevé's classical theorem states that sets of zero length are removable for bounded analytic functions. The result is false for sets of finite, positive length. Iwaniec and Martin conjectured a counterpart for quasiregular mappings, that in $\mathbb{R}^{n}$ sets of $s$-dimensional Hausdorff measure zero, $s=n /(K+1)$, are removable for bounded $K$-quasiregular mappings. In two dimensions I later showed the removability for sets of dimension $\operatorname{dim}(E)<2 /(K+1)$, and that for every $s>2 /(K+1)$ there is a nonremovable set of dimension $s$.
In the present work we study the borderline case in two dimensions. It turns out, perhaps surprisingly, that there is an improved version of Painleve's theorem: When $K>1$, all planar sets of sigma-finite $2 /(K+1)$-dimensional measure are removable for bounded $K$ quasiregular mappings.

SAturday, May 14, 11:45 AM - 12:10 PM

# POLYNOMIAL CONTROL FOR LINEAR SYSTEMS AND ZEROS OF ENTIRE FUNCTIONS 

Roger Barnard

Texas Tech University

We will study the problem of controlling a linear system $x^{\prime}=A x+b u$ with polynomial controls. We discuss conditions on the eigenvalues of $A$ that determine when the system is controllable with a polynomial of degree less than or equal to $n$ or $n-1$. We verify the conjecture that a certain entire function has no zeros of multiplicity $n+1$, hence proving that if $A$ has a certain single Jordan block form then the system is controllable with a polynomial of degree $n$.

Saturday, May 14, 2:00 PM $-2: 15$ PM

# SOME MEMORIES OF A MEMORABLE THESIS STUDENT: AL BAERNSTEIN IN MADISON, 1966-68 

Daniel Shea<br>University of Wisconsin, Madison

We recall how we acquired Al as a thesis student, then how we found our way together to some problems he liked after trying some he didn't. Eventually there are Eureka moments and solutions emerge: but the familiar process had some unusual features, as you might expect - if you know Al and his modus operandi over the years.

Saturday, May 14, 2:45 PM - 3:10 PM

## ALBERT BAERNSTEIN II'S CONTRIBUTIONS TO FUNCTION THEORY

Daniel Girela<br>Universidad de Málaga

In this talk we shall present some of the most relevant contributions of Al Baernstein to function theory. We shall start introducing the $\star$-function in the plane and show how it can be used to solve extremal problems. In particular, we shall point out the use of $\star$-functions to prove that the Koebe function is extremal for a very large class of problems
about integral means in the class $S$ of univalent functions, to prove the spread relation, and to obtain sharp $L^{p}$-inequalities for conjugate functions.

We shall also discuss other contributions of Al to the theory of conformal mappings and recall his work on BMOA-functions and, if time is left, we shall finish speaking a little bit about his work related to Bloch and Landau constants.

Saturday, May 14, 3:20 PM - 3:45 PM

## ALBERT BAERNSTEIN II'S CONTRIBUTIONS TO SYMMETRIZATION, PART I

Carlo Morpurgo<br>University of Missouri, Columbia

This talk is about the method of polarization, or "two-point symmetrization", as applied to integral inequalities of all sorts. This technique was most effectively developed by Baernstein and Taylor in 1976, and it was refined and extended by Al himself, his students and others over the years. We will review these developments, and we will show how the method has impacted, and continues to impact, the work of several mathematicians.

## Saturday, May 14, 3:55 PM - 4:20 PM <br> ALBERT BAERNSTEIN II'S CONTRIBUTIONS TO SYMMETRIZATION, PART II

Almut Burchard<br>University of Virginia

Al Baernstein's work on polarization has been extensively cited and has influenced many researchers. His contributions to rearrangement inequalities for which polarization fails may be less widely known. We will discuss several of Al's results in this direction, including Riesz' inequality for the simultaneous symmetrization of three functions on the circle, and an inequality for k -plane transforms obtained jointly with Michael Loss.

## ABSTRACTS OF POSTERS

## HARDY SPACE THEORY FOR UNBOUNDED SINGULAR INTEGRALS

Ryan Berndt<br>Ohio State University

We examine singular integral operators where the kernel is allowed to have singularities that are not allowed in the Calderón-Zygmund theory. These operators are not, in general, bounded on $L^{2}(\mathbb{R})$, yet there is a Hardy space theory for them.

# BLOW-UP AND CRITICAL EXPONENTS FOR QUASILINEAR PARABOLIC SYSTEMS 

Konstantin O. Besov<br>Steklov Mathematical Institute, Russian Academy of Sciences<br>E-mail: kbesov@mi.ras.ru

We consider the Cauchy problem for systems of inequalities of the form

$$
\begin{cases}u_{t}-A_{1}(t, x, u, \nabla u) \geq b_{1}(t, x) u^{P} v^{Q}+f_{1}(t, x) & \text { in } \mathbb{R}_{+} \times \mathbb{R}^{N}, \\ v_{t}-A_{2}(t, x, v, \nabla v) \geq b_{2}(t, x) u^{R} v^{S}+f_{2}(t, x) & \text { in } \mathbb{R}_{+} \times \mathbb{R}^{N},\end{cases}
$$

where $u=u(t, x) \geq 0, v=v(t, x) \geq 0, A_{i}$ are "positive" operators in the divergence form (e.g., Laplacian or $p$-Laplacian), $b_{i}(t, x), f_{i}(t, x) \geq 0$, and $P, Q, R, S \geq 0$.

Under additional assumptions concerning the functions $A_{i}$, we find sufficient conditions for the nonexistence of nontrivial nonnegative weak solutions to such systems with nonnegative initial data ( $u_{0}, v_{0}$ ). In simple cases, these sufficient conditions are given by restrictions on the exponents $P, Q, R, S$ and sometimes coincide with necessary conditions. We also obtain a priori estimates and estimates of the lifespan of solutions in terms of the behavior of initial data and the functions $b_{i}$ and $f_{i}$.

The proofs are based on the method of nonlinear capacity (integral relations) proposed and developed by E. Mitidieri and S. Pohozaev.

# WEIGHTED HARDY INEQUALITIES 

Ritva Hurri-Syrjänen

University of Helsinki
(Joint work with David E. Edmunds)

For bounded Lipschitz domains $D$ in $\mathbb{R}^{n}$ it is known that if $1<p<\infty$, then for all $\beta \in\left[0, \beta_{0}\right.$ ), where $\beta_{0}=p-1>0$, there is a constant $c<\infty$ with

$$
\int_{D}|u(x)|^{p} \operatorname{dist}(x, \partial D)^{\beta-p} d x \leq c \int_{D}|\nabla u(x)|^{p} \operatorname{dist}(x, \partial D)^{\beta} d x
$$

for all $u \in C_{0}^{\infty}(D)$. We show that if $D$ is merely assumed to be a bounded domain in $\mathbb{R}^{n}$ that satisfies a Whitney cube-counting condition with exponent $\lambda$ and has plump complement, then the same inequality holds with $\beta_{0}$ now taken to be $\frac{p(n-\lambda)(n+p)}{n(p+2 n)}$. Further, we extend the known results of H. Brezis and M. Marcus (1998), M. and T. Hoffman-Ostenhof and A. Laptev (2002) and J. Tidblom (2004) concerning the improved Hardy inequality

$$
\int_{D}|u(x)|^{p} \operatorname{dist}(x, \partial D)^{-p} d x+|D|^{-p / n} \int_{D}|u(x)|^{p} d x \leq c \int_{D}|\nabla u(x)|^{p} d x
$$

$c=c(n, p)$, by showing that the class of domains for which the inequality holds is larger than that of all bounded convex domains.

# EXISTENCE, UNIQUENESS AND REGULARITY OF THE FREE BOUNDARY IN THE HELE-SHAW PROBLEM WITH A DEGENERATE PHASE 

Marianne K. Korten<br>Kansas State University<br>(Joint work with Ivan Blank and Charles N. Moore)

The Hele-Shaw model describes the flow of a viscous fluid being injected through the slot G between two nearby plates. It is used in injection molding for the production of packaging materials and the interior plastic parts of cars and airplanes, in electromechanical machining, and to study the diffusion of nutrients and medicines within certain tumors.

We obtain the unique weak solution $\left(u^{\infty}, V\right)$ to the Hele-Shaw problem with a mushy zone $0<u^{\infty}<1$,

$$
u_{t}^{\infty}=\Delta V \text { in } G^{c} \times(0,+\infty), u^{\infty}(x, 0)=u_{I}(x), V(x, t)=p(x) \text { in } \partial G \times(0,+\infty),
$$

as the (pointwise) "Mesa" type limit of $\left(u^{(m)}, m\left(u^{(m)}-1\right)_{+}\right)$, where the $u^{(m)}$ are solutions to one-phase Stefan problems with increasing diffusivities, $u_{t}^{(m)}=\Delta\left(u^{(m)}-1\right)+$, with fixed
initial and boundary data $0 \leq u_{I} \leq 1$ and $p(x)>0$. The slot $G \subset \mathbb{R}^{n}$ does not need to be connected. The main tools are information available from Korten's earlier work about solutions and interphases in the one-phase Stefan problem, Blank's results on the obstacle problem, and new energy estimates for Stefan problems due to Moore and Korten.

We obtain the traditional formulation (by means of the Baiocchi transformation) as an obstacle problem. At this point regularity in space follows from the work of Blank and Caffarelli.

# QUASICONFORMAL MAPPINGS WITH CONVEX POTENTIALS 

Leonid V. Kovalev<br>Washington University in Saint Louis<br>(Joint work with Diego Maldonado)

Given a differentiable convex function $u: \mathbb{R}^{n} \rightarrow \mathbb{R}$, one can consider its gradient $\nabla u$ as a mapping from $\mathbb{R}^{n}$ to itself. Sometimes this mapping turns out to be quasiconformal. Recall that $f \in W_{\mathrm{loc}}^{1, n}\left(\mathbb{R}^{n} ; \mathbb{R}^{n}\right)(n \geq 2)$ is quasiconformal if $\operatorname{det} D f \approx\|D f\|^{n}$ a.e. The class of quasiconformal mappings with convex potentials is not very small. For example, given a set $E \subset \mathbb{R}^{n}$ of Hausdorff dimension less than 1 , one can find a convex function $u$ such that $\nabla u$ is quasiconformal, and $\operatorname{det} D^{2} u=0\left(\right.$ or $\left.\operatorname{det} D^{2} u=\infty\right)$ at every point of $E$. This is not the only reason why such mappings are interesting.

## APPROXIMATION AND SPANNING IN HARDY SPACE, BY AFFINE SYSTEMS

Richard S. Laugesen<br>University of Illinois, Urbana-Champaign<br>(Joint work with Huy-Qui Bui, University of Canterbury)

We seek to span the Hardy space $H^{1}(\mathbb{R})$ with a small scale affine system $\left\{\eta\left(a_{j} x-k\right)\right.$ : $j>0, k \in \mathbb{Z}\}$. Here the $a_{j}$ are given positive real numbers that increase to infinity.

Spanning seems a plausible property even for generic $\eta \in H^{1}(\mathbb{R})$. Such a broad conjecture is currently out of reach, but we prove spanning for the large class of $\eta$ having the "difference" form $\eta(x)=\psi(x)-\psi(x-1)$ for some $\psi \in L^{1}(\mathbb{R})$ with $\psi^{\prime} \in H^{1}(\mathbb{R})$ and $\int_{\mathbb{R}} \psi(x) d x \neq 0$. (For example, $\psi$ can be any Schwartz function with nonzero integral.)

Spanning is proved by means of explicit approximation formulas.

# SEMIGROUPS AND SEMIHYPERGROUPS: SIMILARITIES AND CONTRASTS 

Norbert Youmbi

University of South Florida

A semihypergroup is roughly speaking a locally compact Hausdorff space which has enough structure so that a convolution on the corresponding vector space of Radon measures makes it a Banach algebra. Dunkl called it a hypergroup (without involution) while Jewett referred to it as a semiconvo. Semihypergroups generalize in many ways locally compact semigroups. Several results for semigroups also hold for semihypergroups, but there still exists striking contrast between the two concepts. The aim of this presentation is to highlight those points with interesting examples.

