Abstract

The point-wise product $b \cdot h$ of functions $b \in BMO(\mathbb{R}^n)$ and $h \in H^1(\mathbb{R}^n)$ need not be locally integrable. However, in view of the duality between $BMO$ and $H^1$, we are able to give a meaning to $b \cdot h$ as a Schwartz distribution, denoted by $b \times h \in \mathcal{D}'(\mathbb{R}^n)$. The central question is concerned with the regularity of $b \times h \in \mathcal{D}'(\mathbb{R}^n)$. We prove a decomposition:

$$b \times h = \alpha + \beta,$$

where $\alpha$ is a function in $L^1(\mathbb{R}^n)$ while $\beta$ is a distribution in a Hardy-Orlicz space. Precisely this means that its maximal function $M\beta$ satisfies

$$\int_{\mathbb{R}^n} \frac{M\beta}{\log(e + M\beta)} \, d\mu < \infty,$$

where $d\mu = \frac{dx}{\log(e + |x|)}$

The Jacobian determinants and more general div-curl products come to a play as atoms.