Mathematics 131

Exam 1

4 February 2014

Directions: This exam should consist of eighteen questions: the first 16 are worth 5 points apiece, and the last two questions are worth 10 points apiece. Write your name and student ID number at the top of this page. Show your work on each problem.

You may not use a calculator or any written aids.

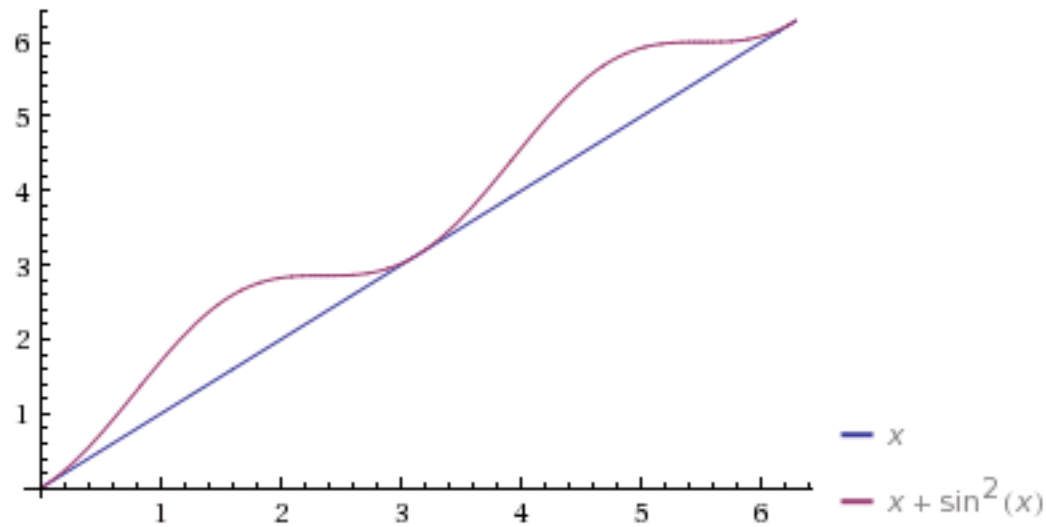
This space for grading purposes only. Do not write here.

Problem 1:	Problem 10:
Problem 2:	Problem 11:
Problem 3:	Problem 12:
Problem 4:	Problem 13:
Problem 5:	Problem 14:
Problem 6:	Problem 15:
Problem 7:	Problem 16:
Problem 8:	Problem 17:
Problem 9:	Problem 18:

Problem 1: Consider the function $\sqrt{x^4 - 1}$. What is its domain?

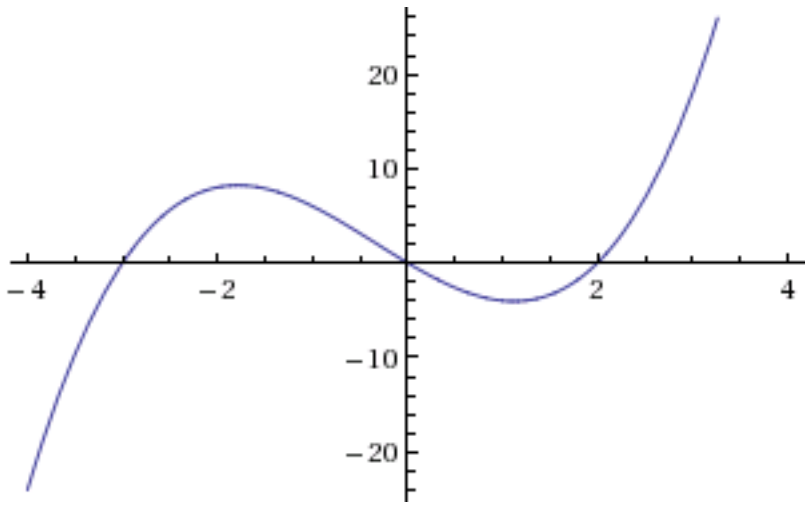
Problem 2: Consider the function $\sqrt{(x+1)(2-x)}$. What is its domain?

Problem 3: The image below consists of the graphs of $y = x + \sin^2 x$ and $y = x$ on the interval $[0, 2\pi]$. The graph of $y = x + \sin^2 x$ lies above the graph of $y = x$ except at three points where the curves intersect, $(0, 0)$, (π, π). and $(2\pi, 2\pi)$. Draw the graph of the inverse function of $x + \sin^2 x$ on the graph. (Hint: don't try to come up with a symbolic formula for the inverse function.)



Problem 4: Is $x^4 + 1$ an even function, an odd function, or neither? Explain your answer.

Problem 5: Below is the graph of a function $y = f(x)$. Draw the graph of the function $f(2x)$ on the same graph.



Problem 6: Let $f(x)$ be a real-valued function with domain $(-\infty, \infty)$. Suppose we start with $f(x)$ and create a new function $g(x)$ by manipulating the graph of $f(x)$ with the following actions. We perform them in the order in which they are listed. First, compress the graph horizontally by a factor of 4; second, shift the graph up by 1; third, stretch the graph vertically by a factor of 2. What is the formula for $g(x)$ in terms of $f(x)$?

Problem 7: Let

$$G(x) = \begin{cases} -1 & \text{if } x \geq 0 \\ -x^2 - 1 & \text{if } x < 0 \end{cases},$$

Compute $\lim_{x \to 0^+} G(x)$, $\lim_{x \to 0^-} G(x)$, and $G(0)$ to show that all three of them are equal.

Problem 8: Compute the following limit or explain why it does not exist:

$$\lim_{x \to 2^+} \frac{x^2 - 2x + 9}{x + 1}.$$

Problem 9: Compute the following limit or explain why it doesn't not exist:

$$\lim_{x \to -1} \frac{x - x^3}{x + 1}.$$

Problem 10: Compute the following limit or explain why it does not exist:

$$\lim_{x \to 2^-} \frac{\sqrt{x} - \sqrt{2}}{x - 2}.$$

Problem 11: Compute the following limit or explain why it does not exist.

$$\lim_{x \to 1^+} \frac{x^2 - 1}{(x-1)^2}$$

Problem 12: Draw the graph of a function $f(x)$ which is defined for all real numbers and has the following properties: $\lim_{x \to 1^+} f(x)$ exists, $\lim_{x \to 1^-} f(x)$ exists, and

$$\lim_{x \to 1^+} f(x) > f(1) > \lim_{x \to 1^-} f(x).$$

(Hint: you don't have to give a formula for $f(x)$, just the graph.)

Problem 13: We talked in class about the function $\sin \frac{1}{x}$. Why doesn't $\lim_{x \to 0^+} \sin \frac{1}{x}$ exist?

Problem 14: Consider the function

$$G(x) = \begin{cases} -1 & \text{if } x \geq 0 \\ x^2 + 1 & \text{if } x < 0 \end{cases}.$$

Explain why $\lim_{x \to 0} G(x)$ doesn't exist. However, show that $\lim_{x \to 0} (G(x))^2 = 1$. (Hint: for the second part, what is $(G(x))^2$ when $x \geq 0$? When $x < 0$?)

Problem 15: Consider the following statement: If $\lim_{x\to 0} f(x)$ exists and $f(0) \neq 0$, then $\lim\limits_{x\to 0} \dfrac{1}{f(x)}$ exists. Explain why this statement is incorrect.

Problem 16: Write a poem, draw a picture, tell a joke, or state something you're passionate about (besides math, of course).

Problem 17: Can you give an example of a function $f(x) = a^x$ with the property that translating its graph to the left by 3 units corresponds to a vertical stretch by 8 units? (Hint: if you are confused about the letter a here, try working with the function 3^x and then write down the formulas that correspond to shifting left and stretching by 8. These formulas won't be the same for 3^x, but can you find a different exponential function for which the two formulas are equal?)

Problem 18: Suppose that $f(x)$ is a function whose domain is all of the real numbers. Suppose that f is also continuous at every real number. Below is a table which gives the values of $f(x)$ for certain values of x.

x	-1	1	3	4	5	7
$f(x)$	-2	1	2	-3	1	3

How many zeros must this function f have? Be sure to state explicitly the names of any theorems you use.