Mathematics 131

Exam 2

4 March 2014

Directions: This exam should consist of eighteen questions: the first 15 are worth 5 points apiece, and the last three questions are worth 10 points apiece, for a total of 105 points. Make sure your name and student ID number appear on this exam. Show your work on each problem.

You may not use a calculator or any written aids.

This space for grading purposes only. Do not write here.

Problem 1:	Problem 10:
Problem 2:	Problem 11:
Problem 3:	Problem 12:
Problem 4:	Problem 13:
Problem 5:	Problem 14:
Problem 6:	Problem 15:
Problem 7:	Problem 16:
Problem 8:	Problem 17:
Problem 9:	Problem 18:

Problem 1: List any horizontal, vertical, or oblique asymptotes the following function has; if it doesn't have an asymptote of a particular type, explain why not.

$$\frac{8x^3 + x^2 - 2x + 1}{x^2 + 4}$$

Problem 2: What is the derivative of $5x^3 + 3x^2 + 1$?

Problem 3: Suppose that $f(x) = x\sqrt{x+1}$. Write down a limit which gives the derivative of f at $x = 1$. You do not need to compute this limit.

Problem 4: Compute the following limit:

$$\lim_{x\to\infty} \sqrt{x^2 + 4x + 1} - x.$$

(Note: I did an example almost identical to this in class, so you might consider this a gentle nudge to attend class regularly.)

Problem 5: Suppose that $f(x)$ is a one-to-one function which is differentiable everywhere. Use the following table of data to give an equation for the tangent line to $f^{-1}(x)$ at $x = 2$.

x	1	2	3	4	5
$f(x)$	5	4	3	2	1
$f'(x)$	-2	-4	-5	-7	-11

Problem 6: If $f(x) = 2^{x \tan x}$, what is $f'(\pi)$?

Problem 7: Suppose that $f(x) = \sqrt{x+1}$ and the tangent line to $g(x)$ at $x = 0$ is given by $y = 2x + 3$. What is the derivative of $f(g(x))$ at $x = 0$?

Problem 8: For which values of a is the following function continuous? If f is not continuous for any value of a, explain why.

$$f(x) = \begin{cases} ax + \frac{1}{ax} & \text{if } x \geq 1 \\ -x^2 + ax & \text{if } x < 1 \end{cases} .$$

Problem 9: Suppose that $f(x) = \begin{cases} \cos x - 1 & \text{if } x \geq 0 \\ \sin x & \text{if } x < 0 \end{cases}$.

1. Explain why f is continuous at $x = 0$.
2. Explain why f is not differentiable at $x = 0$.

Problem 10: Suppose that $f(x) = \frac{x}{x^2+1}$. For which values of x is $f'(x) = 0$?

Problem 11: Explain why

$$\lim_{x\to\infty} \frac{x^3 + 3x^2 + x^2 \sin x}{x^5 + 1} = 0.$$

Problem 12: Consider the curve given by $y^3 + x^2y^2 + x - 1 = 0$. Write an equation for the tangent line to this curve at the point $(1, -1)$.

Problem 13: Give an example of a function which has two vertical asymptotes and an oblique asymptote. You can either write down a formula or draw a picture.

Problem 14: Consider the function $f(x) = x^2 \sec x + x$. What is the slope of the tangent line to f at $x = \frac{\pi}{4}$? You do not need to simplify your answer.

Problem 15: Write a poem, tell a dream, give a joke, or draw a meme.

Problem 16: Consider the function $f(x) = x\sqrt{x+1}$.

1. Compute $f'(x)$.
2. Compute the equation for the tangent line to $f(x)$ at $x = 3$
3. Use the tangent line to approximate the value of $f(3.1)$.
4. Do you think your approximation is an overestimate or an underestimate of the true value of $f(3.1)$?

Problem 17:

1. Use long division to show that

$$\frac{u^5 - 1}{u - 1} = u^4 + u^3 + u^2 + u + 1$$

   and conclude that $(u-1)(u^4 + u^3 + u^2 + u + 1) = u^5 - 1$.

2. Plug in $u = x^{1/5}$ and use the above to compute the following limit:

$$\lim_{x \to 1} \frac{x^{1/5} - 1}{x - 1}.$$

Problem 18: Suppose that f and g are two differentiable functions.

1. Write the formula for the derivative of $\frac{f(x)}{g(x)}$ when $g(x) \neq 0$, i.e. write the quotient rule.

2. Using just the product rule and chain rule, explain why the quotient rule is true. (Hint: write $\frac{f(x)}{g(x)} = f(x) \cdot \frac{1}{g(x)}$.)

3. Using only the quotient rule and the fact that $(\sin x)' = \cos x$ and $(\cos x)' = -\sin x$, explain why $(\cot x)' = -\csc^2 x$. If you can apply the quotient rule correctly but cannot simplify your answer to $-\csc^2 x$, partial credit will be given. (Hint: write $\cot x = \frac{\cos x}{\sin x}$.)