Mathematics 131

Exam 3

8 April 2014

Directions: This exam should consist of seventeen questions: the first 15 are worth 5 points apiece, and the last two questions are worth 15 points apiece, for a total of 105 points. Make sure your name and student ID number appear on this exam. Show your work on each problem.

You may not use a calculator or any written aids.

This space for grading purposes only. Do not write here.

Problem 1:	Problem 10:
Problem 2:	Problem 11:
Problem 3:	Problem 12:
Problem 4:	Problem 13:
Problem 5:	Problem 14:
Problem 6:	Problem 15:
Problem 7:	Problem 16:
Problem 8:	Problem 17:
Problem 9:	Problem 18:

Problem 1: Compute the derivative of x^x. (Hint: use logarithmic differentiation.)

Problem 2: Compute the derivative of $\log_2(\arctan x)$.

Problem 3: Suppose that a particle moves along the circle $x^2 + y^2 = 1$. Thinking of x and y differentiable functions of t,

1. (3 points) Solve for $\frac{dx}{dt}$ in terms of x, y, and $\frac{dy}{dt}$.
2. (2 points) What is $\frac{dx}{dt}$ at the point $(1, 0)$?

Problem 4: Suppose you find yourself in the following real world situation: in front of you are several cans of your preferred beverage (please drink responsibly) and you are asked how many cans can be poured into a standard frisbee. Imagining the can and frisbee as cylinders, you measure the circular top of the can to have a radius of 1 inch and a height of 5 inches. The frisbee has a radius of 5 inches and a height of 1 inch.

1. Assuming the volume formula of $\pi r^2 h$ for a cylinder, how many cans of your preferred beverage can be poured into the frisbee?
2. Suppose your ruler is only good to .1 inches (i.e. $\Delta r = .1 = \Delta h$). How does this affect your measurement of the volume of the frisbee?

Problem 5: Consider the function $f(x) = x^2 - 12x + 12$. Compute the co-ordinates of the absolute maximum and absolute minimum of $f(x)$ on the interval $[0, 10]$.

Problem 6: Suppose that $f'(x) = \frac{x^3 - 3x^2}{x^2 - 1}$. How many critical values does f have? Be sure to justify your answer. NOTE: the formula is for $f'(x)$ not $f(x)$.

Problem 7: Draw the graph of a function which has exactly three critical values in the interval $[-2, 2]$ but only two of them are local extrema.

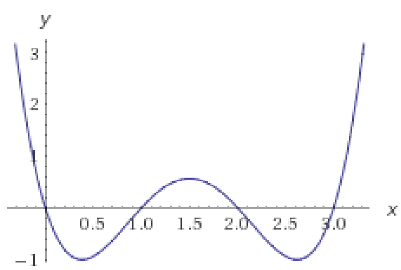
Problem 8: Consider the function $f(x) = x^2$ on the interval $[1, 3]$.

1. Determine X so that the following sentence is correct: The Mean Value Theorem applied to x^2 on the interval $[1, 3]$ says that there is a c in $(1, 3)$ so that $f'(c) = X$

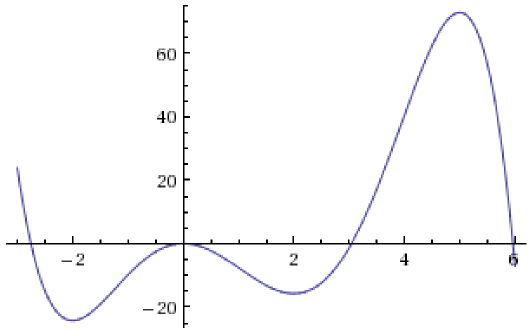
2. Find a c in $(1, 3)$ so that $f'(c) = X$, where X is your answer to the previous part.

Problem 9: Suppose that $f''(t) = t + 1$ and that $f'(0) = 1$ and $f(0) = 2$. Give a formula for $f(t)$.

Problem 10: Suppose that the graph of $f'(x)$ is given below. On what interval(s) is $f(x)$ decreasing? NOTE: the graph is of $f'(x)$ not $f(x)$.



Problem 11: Suppose that the graph of $f'(x)$ is given below. On what interval(s) is $f(x)$ concave up? NOTE: the graph is of $f'(x)$ not $f(x)$.



Problem 12: Suppose that $f'(x) = (x-2)(x+3)$. On which intervals is f decreasing? NOTE: the formula is for $f'(x)$ not $f(x)$.

Problem 13: Suppose that $f(x) = x^3 + 2x^2 + x + 1$. On which interval(s) is f concave up?

Problem 14: Compute the following limit:

$$\lim_{x \to 0} \frac{1 - \frac{x^2}{2} - \cos x}{x^4}.$$

Problem 15: What is the funniest, most interesting, or coolest thing you have read, heard, learned, or done this year? (It need not be calculus related.)

Problem 16: Suppose that $f(x) = x^3 - 3x + 3$,

(a) (3 points) Compute $f'(x)$ and $f''(x)$.

(b) (3 points) Identify the local extrema of f, if there are any.

(c) (3 points) Identify the inflection inflection points of f, if there are any.

(d) (3 points) Sketch the graph of f, labeling your axes appropriately — be sure to label the co-ordinates where $f(x)$ crosses the x and y axes.

(e) (3 points) Explain why this function has a zero in the interval $(-3, -2)$. (Hint: use the Intermediate Value Theorem.)

Problem 17: Suppose that $f(x) = \frac{x^2}{x^2+1}$.

(a) (3 points) Show that $f'(x) = \frac{2x}{(x^2+1)^2}$ and $f''(x) = \frac{2-6x^2}{(x^2+1)^3}$. (Even if you cannot get this part, you may use these formulas for the rest of the problem.)

(b) (3 points) Identify the local extrema of f, if there are any.

(c) (3 points) Identify the inflection inflection points of f, if there are any.

(d) (3 points) Determine all asymptotes of $f(x)$, if there are any.

(e) (3 points) Sketch the graph of f, labeling your axes appropriately — be sure to label the co-ordinates of any points where $f(x)$ crosses the x and y axes.