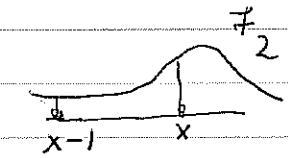
<u>Return to Convolutions</u> Suppose X_1, X_2 are indep Cts RV's w/ pdf's f_1, f_2. Let h = pdf of $X_1 + X_2$

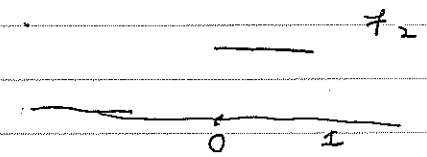
Then $$h(x) = \int_{-\infty}^{\infty} f_1(y)\, f_2(x-y)\, dy = f_1 * f_2\,(x)$$

Suppose $f_1(y) = 1$, $0 < y < 1$. Then

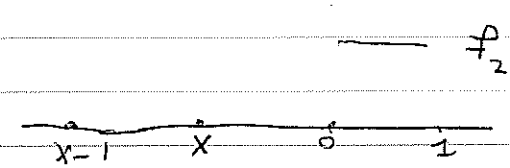


$$* \quad h(x) = \int_0^1 f_2(x-y)\,dy = \int_{x-1}^{x} f_2(z)\,dz$$

Now Suppose $f_2(y) = 1$, $0 < y < 1$ also.



Then h can be computed as follows: start with large negative x.

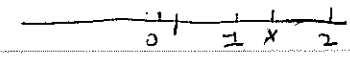


$f_2(y) \equiv 0$ for $x-1 < y < x$

So $h(x) = 0$, $x < 0$

Next, look at $0 < x < 1$

$$\int_{x-1}^{x} f_2(y)\,dy = \int_0^x 1\,dy = x$$



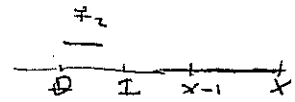
$h(x) = x$, $0 \le x \le 1$

Next, look at $1 \le x \le 2$

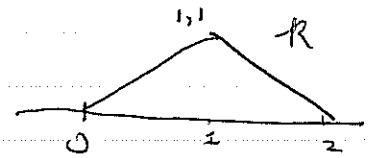
$$\int_{x-1}^{x} f_2(y)\,dy = \int_{x-1}^{1} dy = 1 - (x-1) = 2 - x$$

$h(x) = 2 - x$, $1 \le x \le 2$

For $x \geq 2$, $k(x) = 0$. So



$$k(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \\ 0, & x \geq 2 \end{cases}$$



Let X_1, X_2, X_3 be indep $U(0,1)$'s. Let

$Y = \sum_1^3 X_i$, g = pdf of Y. Then

$g = k * f_1$, where f_1 = the unito pdf on $[0,1]$ and k = pdf of $X_1 + X_2$

$$g(x) = \int_{x-1}^{x} k(y)\, dy$$

Start w/ $x < 0$. $g(x) = 0$, $x < 0$

Move x forward. $g(x) = 0$, $x > 3$

Say $x \in (2,3)$. $g(x) = \int_{x-1}^{2} k(y)\, dy = \int_{x-1}^{2} (2-y)\, dy$

$$= -\tfrac{1}{2}(2-y)^2 \Big|_{y=x-1}^{y=2} = \tfrac{1}{2}\left(2-(x-1)\right)^2$$

$$= \tfrac{1}{2}(3-x)^2$$

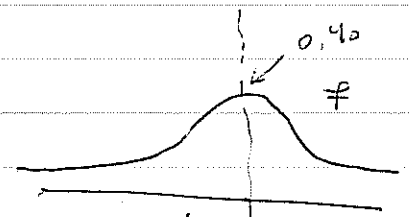
Now do $0 \leq x \leq 2$

<u>Next job</u> Let f be pdf of X, any ctns r.v., and h be pdf of $aX = Z$. Find pdf h of Z. Here $a > 0$

$$F_Z(z) = P(Z \le z) = P(aX \le z) = P(X \le z/a) = F_X(z/a)$$

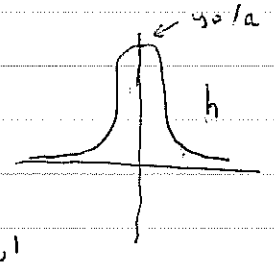
$$h(z) = \frac{1}{a} f(x/a)$$

e.g., if f looks like

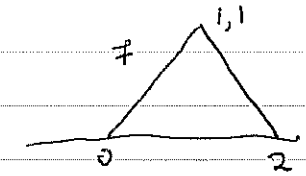


and a is small, then

h looks like

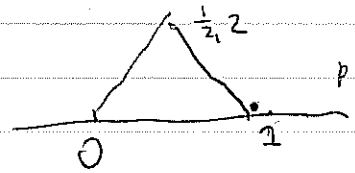


<u>For tent fcn</u>



pdf of $X_1 + X_2$

$$h(x) = 2f(2x)$$



pdf of $\dfrac{X_1 + X_2}{2}$