Covariance Problems

(1) Suppose that X, Y are RV's with $\operatorname{Var} X = 9$, $\operatorname{Var} Y = 4$, and $\rho(X,Y) = -\frac{1}{6}$. Find

(a) $\operatorname{Var}(X+Y)$, (b) $\operatorname{Var}(X-3Y+4)$, (c) $\operatorname{Cov}(X,Y)$

(2) Suppose that X, Y, Z are RV's with

$\operatorname{Var} X = 1$, $\operatorname{Var} Y = 4$, $\operatorname{Var} Z = 8$,

$\operatorname{Cov}(X,Z) = -1$, $\operatorname{Cov}(X,Y) = 1$, $\operatorname{Cov}(Y,Z) = 2$. Find

(a) $\operatorname{Cov}(X+Y, X+Z)$ (b) $\operatorname{Var}(3X-Y-2Z+1)$

(3) Suppose that $X_1, \ldots, X_n$ are RV's s.t. the variance of each RV is 1 and the correlation between each pair of different variables is $1/4$. Find

$\operatorname{Var}\left(\sum_{i=1}^{n} X_i\right)$.

(4) If $\operatorname{Var} X = 9$, $\operatorname{Var} Y = 4$, use Schwarz's inequality to find the largest and smallest possible values of $\operatorname{Cov}(X,Y)$.

⑤ Suppose that discrete RV's have p.f. specified by the table

X \ Y	4	5	6
1	.3	.2	.1
2	.2	.1	.1

Find $E(XY)$ and $Cov(X,Y)$ and $\rho(X,Y)$.

Answers ① (a) 11, (b) 51, (c) −1

② (a) 3, (b) 59

③ $n + \frac{n(n-1)}{4}$

④ largest = 6, smallest = −6

⑤ $E(XY) = 6.6$, $EX = 1.4$, $EY = 4.7$

$Cov(X,Y) = .02$

$E(X^2) = 2.2$, $Var\ X = .24$

$E(Y^2) = 22.7$, $Var\ Y = .61$

$\rho(X,Y) = .052$