Exam 1

① (a) $P(2 \le X \le 5) = P(3) + P(5) = .5 + .1 = \boxed{.6}$

(b) $F(4) = P(X \le 4) = P(1) + P(3) = \boxed{.9}$

(c) $EX = 1 \cdot P(1) + 3 \cdot P(3) + 5 \cdot P(5) = \boxed{2.4}$

(d) $E\left(1/X\right) = \frac{1}{1} P(1) + \frac{1}{3} P(3) + \frac{1}{5} P(5)$

$= .587$

(2) 4R, 7B, 9G. Draw 3 wo repl. Total balls = 20

(a) $P(3 \text{ different colors}) = \frac{4}{20}\,\frac{7}{19}\,\frac{9}{18}\,3! = .22105 = \frac{1512}{6840}$

(b) $P(2 \text{ reds}) = P(RR \text{ other}) \cdot 3 = \left(\frac{4}{20}\,\frac{3}{19}\,\frac{16}{18}\right) * 3 = 21/95$

$= 576/6840 = .0842 = 8/95$

(3) $P(G) = .3$, $P(S) = .7$, $P(M \mid G) = .55$, $P(M \mid S) = .2$

(a) $P(M) = P(G)P(M \mid G) + P(S)P(M \mid S)$

$= (.3)(.55) + (.7)(.2) = .165 + .14 = .305$

(b) $P(G \mid M) = P(M \mid G)\,P(G) / P(M)$

$$= \frac{.55\ (.3)}{.305} = \frac{.165}{.305} = .54$$

(4) ~~V = visit, B = bowls, P(V) = .7, P(B) = .25, P(B|V) = .2~~

(a) $P(B \cup V) = P(B) + P(V) - P(BV)$

$P(BV) = P(B|V)\,P(V) = (.2)(.7) = .14$

$P(B \cup V) = .25 + .7 - .14 = .81$

(b) $P(V \mid (B \cup V)) = \dfrac{P(V \cap (B \cup V))}{P(B \cup V)} = \dfrac{P(V)}{P(B \cup V)} = \dfrac{.7}{.81} = \dfrac{70}{81}$

$= .864$

1, 2 lose

(5) 5, 6 → win. For other x, roll til 6 or $y < x$. If 6, win.

$$P(\text{you win}) = \sum_{x=1}^{6} P(W \mid \text{1st roll is } x)\; P(\text{1st roll is } x)$$

$P(W \mid \text{1st is } x) = 0$, if $x = 1$

$= 0$ if $x = 2$

$= \frac{1}{3}$, $x = 3$ $\left[P(6 \text{ before } 1 \text{ or } 2) = \dfrac{1/6}{\frac{1}{3}+\frac{1}{6}} = \dfrac{1}{3} \right]$

$= \frac{1}{4}$, $x = 4$ $\left[P(6 \text{ before } 1,2 \text{ or } 3) = \dfrac{1/6}{\frac{1}{2}+\frac{1}{6}} = \dfrac{1}{4} \right]$

$= 1$, $x = 5$ or 6

$P(W) = \frac{1}{6}\left[\frac{1}{3} + \frac{1}{4} + 1 + 1\right] = \frac{1}{6}\left[2 + \frac{7}{12}\right] = \frac{1}{6}\left[\frac{31}{12}\right]$

$= 31/72 = .431$

(6) 12 E, 8 F, choose 7. $\binom{20}{7}$ choices in all. $\binom{12}{4}$ to choose E,

$\binom{8}{3}$ to choose F.

$P(4 \text{ E and } 3 \text{ F at bottom}) = \dfrac{\binom{12}{4}\binom{8}{3}}{\binom{20}{7}} = \dfrac{495 \cdot 56}{77{,}520} = .358$

7 (a) # of divisions $= 5^{10} = 9,765,625$

(b) # of divisions $= \binom{10}{3,3,2,2} = \frac{10!}{3!\,3!\,2!\,2!} = 25,200$

8 24 cards, deal 5, 6 in each denomination, 4 denominations

$$P\left(\bigcup_{i}^{4} E_i\right) = \sum P(E_i) - \sum P(E_iE_j) + \sum P(E_iE_jE_k) - P(E_1E_2E_3E_4)$$

$$= 4P(E_1) - 6P(E_1E_2) + 4P(E_1E_2E_3) - P(E_1E_2E_3E_4)$$

$$P(E_1) = P(\text{no jacks}) = \frac{\binom{18}{5}}{\binom{24}{5}}$$

$$P(E_1E_2) = \frac{\binom{12}{5}}{\binom{24}{5}}, \quad P(E_1E_2E_3) = \frac{\binom{6}{5}}{\binom{24}{5}}, \quad P(E_1E_2E_3E_4) = 0$$

$$P\left(\bigcup_{i}^{4} E_i\right) = \frac{1}{\binom{24}{5}}\left\{4\binom{18}{5} - 6\binom{12}{5} + 4\binom{6}{5}\right\}$$

$$= \frac{1}{42504}\left\{34272 - 4752 + 24\right\}$$

$$= \frac{29,544}{42,504} = .69509$$