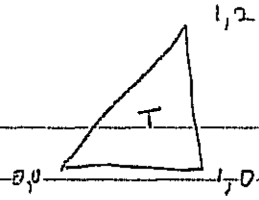
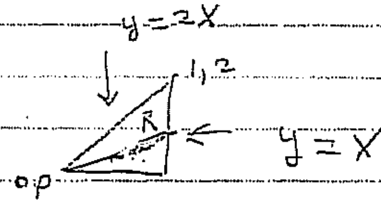
1 $f = \frac{5}{2} x^2 y$ on T



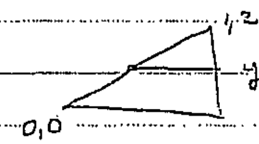


(a) $P(Y \geq X) = \int_R f$

$$= \int_0^1 dx \int_x^{2x} \tfrac{5}{2} x^2 y \, dy$$

$$= \int_0^1 dx \quad \tfrac{5}{4} x^2 y^2 \Big]_x^{2x}$$

$$= \int_0^1 \tfrac{5}{4} x^2 \left[4x^2 - x^2 \right] dx = \int_0^1 \tfrac{15}{4} x^4 \, dx = 3/4$$



(b) $f_Y(y) = \int_{y/2}^1 \frac{5}{2} x^2 y \, dx$

$$= \tfrac{5}{2} y \int_{y/2}^1 x^2 \, dx = \tfrac{5}{2} y \, \tfrac{1}{3} \left(1 - y^3/8\right)$$

$$= 5/6 \, y \left(1 - \tfrac{1}{8} y^3\right), \quad 0 < y < 2$$

$$= 0, \text{ else}$$

(c) in a sufficiently small square containing $(\frac{1}{2}, \frac{1}{2})$ is approximately $5/16$ times the area of the square.

(d) Not independent. Need T to be product set

2 (a) $P(X \le x) = 1 - e^{-\lambda x}$, $x > 0$,

(b) $Y = X^3 - 1$, poss values of $Y = (-1, \infty)$

$P(Y \le y) = P(X^3 - 1 \le y) = P(X^3 \le 1 + y) = P(X \le (1+y)^{1/3})$

$= 1 - e^{-(1+y)^{1/3}\lambda}$

$f_Y(y) = \lambda \frac{d}{dy}(1+y)^{1/3} \cdot e^{-(1+y)^{1/3}\lambda}$

$= \lambda \frac{1}{3}(1+y)^{-2/3} e^{-(1+y)^{1/3}\lambda}$, $-1 < y < \infty$

3 $P(\text{one tosser needs} \ge 5 \text{ tosses}) = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

$= P(\ge 3 \text{ tossers need} \ge 5 \text{ tosses})$

$= \binom{4}{3} \cdot \left(\frac{16}{81}\right)^3 \left(\frac{65}{81}\right) + \left(\frac{16}{81}\right)^4 = .0263$

4 $X \sim N(10, \sigma^2)$. Let $Z = \frac{X-10}{\sigma}$.

$P(X < 15) = P\left(\frac{X-10}{\sigma} < 5/\sigma\right) = \Phi(5/\sigma) = .6$

$5/\sigma = .25$, $\sigma = 5/.25 = 20$, $\sigma^2 = 400$

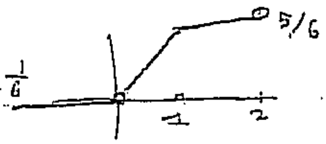
5 $\lambda = np = 200 \cdot (.03) = 6$ $X = \#$ of bumpies

$P(X = 8) \approx e^{-6}\frac{6^8}{8!} = e^{-6}\frac{6^8}{8!} = .103$

6 $f(x) = 12x^2(1-x)$, $0 < x < 1$

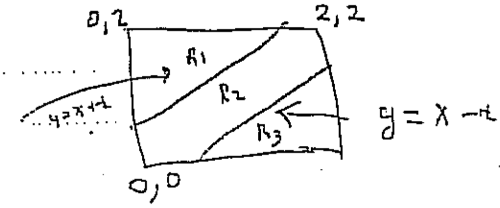
$E(1/X) = \int_0^1 12x(1-x)\,dx = 6 - 4 = 2$

⑦ $P(X=1) = 0, \quad P(X=2) = \frac{1}{6}$



$P(1 < X < 2) = F(2-) - F(1) = \frac{5}{6} - \frac{1}{2} = \frac{1}{3}$

⑧ $y - x \geq t$ or $y - x \leq -t$



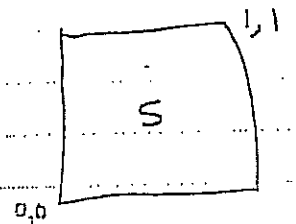
$y \geq x + t$ or $y \leq x - t$

$P(|X-Y| \geq t) = P(R_1) + P(R_3) = 2P(R_1)$

$= 2 \cdot \frac{(2-t)^2}{2 \cdot 4} = \frac{1}{2 \cdot 2}(2-t)^2 = \frac{(2-t)^2}{4}, \quad 0 \leq t \leq 2.$

$t = 1 \Rightarrow P(|X-Y| \geq 1) = \boxed{\frac{1}{4}}$

⑨



$F = \frac{1}{3} xy(2x+y)$

$f(x,y) = \frac{\partial^2}{\partial x \partial y} F = \frac{\partial}{\partial x}\left[\frac{x}{3}(2x+y) + \frac{1}{3}xy\right]$

$= \frac{\partial}{\partial x}\left[\frac{2x^2}{3} + \frac{xy}{3} + \frac{xy}{3}\right]$

$= \frac{4x}{3} + \frac{2}{3}y, \quad (x,y) \in S$

$P(X \leq x) = \lim_{y \to \infty} F(x,y) = F(x,1) = \frac{1}{3}x(2x+1), \quad 0<x<1$

$= \frac{1}{3}(2x^2+x)$

$= 0 \quad$ ~~other~~ $x \leq 0$

$= 1, \quad x \geq 1$

$f_X(x) = \frac{1}{3}(4x+1), \quad 0<x<1$

$= 0$, else