HW 2

1 (11) .28 cigarettes, .10 cigars, .05 both

Take A = smokes cigarettes, B = smokes cigars

(a) $P((A \cup B)^c) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(AB)]$

$= 1 - .28 - .10 + .05 = \boxed{.67}$

(b) $P(B \setminus A) = P(B) - P(BA) = .10 - .05 = \boxed{.05}$

2 (12) $P(S) = .28$, $P(F) = .26$, $P(G) = .20$

$P(SF) = .12$, $P(SG) = .04$, $P(FG) = .06$

$P(SFG) = .02$

(a) $P(\text{none}) = 1 - P(S \cup F \cup G)$

$P(S \cup F \cup G) = P(S) + P(F) + P(G) - P(SF) - P(SG) - P(FG)$

$+ P(SFG) = .74 - .22 + .02 = .54$

$P(\text{none}) = \boxed{.46}$

(c) $P(\text{at least one}) = 1 - P(\text{each takes none}) = 1 - \frac{46}{100} \cdot \frac{45}{99}$

3 (15) (a) S = all 6 element subsets of 52 card deck

A = flushes. There are $\binom{13}{6}$ flushes for each suit.

So $|A| = 4\binom{13}{6}$. also $|S| = \binom{52}{6} = 20,358,520$

$$P(A) = \frac{4\binom{13}{6}}{\binom{52}{6}}$$

3 15 (d) Hand has form $\{a, a, a, b, c, d\}$, with a, b, c, d distinct values

First, count # of hands with $a = ace$, $b = 2$, $c = 3$, $d = 4$

There are $\binom{4}{3}$ ways to pick the aces, and 4 ways each to pick

the 2, 3, 4. So there are $\binom{4}{3} \cdot 4^3$ hands of form $\{ace, ace, ace, 2, 3, 4\}$

2, 3, 4 can be replaced by any 3-element subset of $\{2, \dots, 13\}$.

Then ace can be replaced by any value $\in \{1, 2, \dots, 13\}$, so

$$P(A) = \frac{4^4 \binom{12}{3} \cdot 13}{\binom{52}{6}}$$

Four = b, c, e, see

P. 5

4 28

5R
6 B
8 G

Select 3, w.o. repl. Let S = all 3-element subsets of $\{1, \dots, 19\}$

(a) Let A_1 = all 3-element subsets of $\{1, \dots, 5\}$ (the reds)

A_2 = ———— $\{6, \dots, 11\}$ (the blues)

A_3 = ———— $\{12, \dots, 19\}$ (the greens)

$$P(\text{all balls have same color}) = \frac{\binom{5}{3} + \binom{6}{3} + \binom{8}{3}}{\binom{19}{3}}$$

$$= \frac{5}{19}\,\frac{4}{18}\,\frac{3}{17} + \frac{6}{19}\,\frac{5}{18}\,\frac{4}{17} + \frac{8}{19}\,\frac{7}{18}\,\frac{6}{17}$$

(b) Let B = all $\omega \in S$ with one element in each of $\{1, \dots, 5\}$,

$\{6, \dots, 11\}$, $\{12, \dots, 19\}$. Then

$|B| = 5 \cdot 6 \cdot 8$

$$P(\text{all have different color}) = \frac{|B|}{|S|} = \frac{5 \cdot 6 \cdot 8}{\binom{19}{3}}$$

$$= \frac{5}{19}\,\frac{6}{18}\,\frac{8}{17} \cdot 3!$$

Now assume balls are drawn with replacement.

We'll anticipate later results. Then

$$P(\text{all have same color}) = \left(\frac{5}{19}\right)^3 + \left(\frac{6}{19}\right)^3 + \left(\frac{8}{19}\right)^3$$

$$P(\text{all have different color}) = P(\text{Red then blue then green}) \cdot 3!$$

$$= 3!\,\frac{5 \cdot 6 \cdot 8}{19^3}$$

5 30 Chess club A has 7 players

Club B has 9 players

Choose 4 from each school

(a) $P(\text{Reb is chosen}) = \dfrac{\binom{6}{3}}{\binom{7}{4}} = 4/7$

$P(\text{Elise is chosen}) = 4/9$

$P(\text{person } i \text{ is paired with person } j) = \dfrac{3!}{4!} = \dfrac{1}{4}$

By independence considerations

$$P(R \text{ and } E \text{ are paired}) = \frac{4}{7}\,\frac{4}{9}\,\frac{1}{4} = 4/63$$

5 ~~4~~ (b) P(R and E are chosen not paired)

$= P(R \text{ and } E \text{ are chosen}) - P(R \text{ and } E \text{ are paired})$

$= \frac{4}{7} \cdot \frac{4}{9} - \frac{4}{7}\frac{4}{9}\frac{1}{4} = 12/63$

(c) P(at least one of R and E are chosen)

$= P(R) + P(E) - P(RE)$

$= \frac{4}{7} + \frac{4}{9} - \frac{4}{7}\frac{4}{9} = 16/21$

In (b), (c), we used fact that R and E are independent

HW 2 - Problems 3 and 6

3 again

(a) $P(\text{flush}) = \binom{13}{6} \cdot 4 \cdot \frac{1}{\binom{52}{6}} = .000337$

(b) $P(\text{one pair}) = \binom{13}{1}\binom{4}{2}\binom{12}{4}\binom{4}{1}^4 \frac{1}{\binom{52}{6}} = .4855$

(c) $P(2 \text{ pairs}) = \dfrac{\binom{13}{2}\binom{4}{2}^2\binom{11}{2}\binom{4}{1}^2}{\binom{52}{6}} = .1214$

(d) $P(3 \text{ of a kind}) = \dfrac{\binom{13}{1}\binom{4}{3}\binom{12}{3}\binom{4}{1}^3}{\binom{52}{6}} = .03596$

(e) $P(4 \text{ of a kind}) = \dfrac{\binom{13}{1}\binom{4}{4}\binom{12}{2}\binom{4}{1}^2}{\binom{52}{6}} = .000674$

6 $P(\text{no husband sits next to wife}) =$

$$1 - \left[\binom{5}{1}\frac{9!}{10!} \cdot 2 - \binom{5}{2}\frac{8!}{10!}2^2 + \binom{5}{3}\frac{7!}{10!}2^3 - \binom{5}{4}\frac{6!}{10!}2^4 + \binom{5}{5}\frac{5!}{10!}2^5 \right]$$

$= .3481 = 47/135$

<u>Note</u> In 3, $\binom{52}{6} = 20{,}358{,}520$

(b) P(R and E are chosen but not paired)

$= P(R \text{ and } E \text{ are chosen}) - P(R \text{ and } E \text{ are paired})$

$= \frac{4}{7}\frac{4}{9} - \frac{4}{7}\frac{4}{9}\frac{1}{4} = \frac{16-4}{63} = 12/63$

(c) $P(E \text{ is chosen}, R \text{ is not}) = \frac{4}{7}\frac{5}{9}$

$P(E \text{ is not chosen}, R \text{ is}) = \frac{3}{7}\,4/9$

$P(\text{exactly one is chosen}) = \frac{1}{63}[20+12] = \frac{32}{63}$

Problem 54

Let E_1 = event "hand has no spades"

E_2 = no hearts

E_3 = no diamonds

E_4 = no clubs

Then $P(\text{void in at least one suit}) = P(E_1 \cup E_2 \cup E_3 \cup E_4)$

$$= \sum_{i=1}^{4} P(E_i) - \sum_{1\le i<j\le 4} P(E_iE_j) + \sum_{1\le i<j<k\le 4} P(E_iE_jE_k) - P(E_1E_2E_3E_4)$$

$$= 4P(E_1) - \binom{4}{2}P(E_1E_2) + \binom{4}{3}P(E_1E_2E_3) - P(E_1E_2E_3E_4)$$

Now $P(E_1) = \dfrac{\binom{39}{8}}{\binom{52}{8}}$ [If he has no spades, there are 39 cards for him to choose from]

$$P(E_1E_2) = \frac{\binom{26}{8}}{\binom{52}{8}}$$

$$P(E_1 E_2 E_3) = \frac{\binom{13}{8}}{\binom{52}{8}}$$

$$P(E_1 E_2 E_3 E_4) = 0$$

So $P(\text{void in at least one suit}) =$

$$\frac{1}{\binom{52}{8}} \left\{ 4\binom{39}{8} - 6\binom{26}{8} + \binom{4}{3}\binom{13}{8} \right\}$$

$$\doteq .3146$$