(1) For $2 \le i \le 12$, let A_i be the event "sum $= i$"

and B be the event "each die is ≤ 5"

The sample space consists of all pairs (x, y), where

x and y are integers between 1 and 6. Each (x, y)

occurs with probability $1/36$, so

$$P(B \mid A_i) = \frac{P(BA_i)}{P(A_i)} = \frac{|BA_i|}{|A_i|}$$

Say $i = 7$. Then $|BA_i| = 4$ and $|A_i| = 6$, so

$P(B \mid A_7) = 4/6$ and $P(\text{at least one die} = 6 \mid A_7) = \frac{2}{6}$.

Similarly, $P(\text{at least one die} = 6 \mid A_i)$ equals 0

for $2 \le i \le 6$, equals 1 for $i = 11$ and 12,

and equals $\frac{2}{6}, \frac{2}{5}, \frac{2}{4}, \frac{2}{3}$ for $i = 7, 8, 9, 10$.

② Prob. 19. Let $M=$ proportion of men

$W=$ " " women

$N=$ " going to party

Then $P(N|W)=.6$, $P(N|M)=.37$, $P(M)=.62$

Do (b) first:

$$P(N)=P(N|M)P(M)+P(N|W)P(W)$$

$$=(.37)(.62)+(.6)(.38)=\boxed{.4574}$$

Now do (a).

$$P(W|N)=P(WN)\frac{1}{P(N)}=P(N|W)P(W)\frac{1}{P(N)}$$

$$=(.6)\frac{.38}{.4574}=\boxed{.498}$$

③ Let $A=$ event "coin 1 was chosen" Prob 43

$B=$ event "heads appears"

The coins are equally likely to be chosen, so $P(A)=\frac{1}{3}$.

also, $P(B)=\frac{1}{3}\cdot 1+\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}(.9)=.8$. So

P(2-headed coin was flipped | heads appears)

$$=P(A|B)=\frac{P(AB)}{P(B)}=\frac{P(A)}{P(B)}=\frac{1/3}{(.8)}=\boxed{5/12}$$

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Prob 51. Let G = event "will get job". Then

$P(G|S) = .8, \quad P(G|M) = .4, \quad P(G|W) = .1$

where S = event "will get strong rec", etc. Then

$P(S) = .4, \quad P(M) = .5, \quad P(W) = .1$

(a) $P(G) = P(G|S)P(S) + P(G|M)P(M) + P(G|W)P(W)$

$= (.8)(.4) + (.4)(.5) + (.1)(.1) = .53$

(b) $P(S|G) = \dfrac{P(G|S)\,P(S)}{P(G)} = \dfrac{32}{53} = .604$, strong rec

$P(M|G) = \dfrac{20}{53} = .378$ — moderate rec

$P(W|G) = \dfrac{1}{53} = .019$

$P(S) = P(SG^c) + \cancel{P(SG)}$

(c) $P(S|G^c) = \dfrac{1}{P(G^c)}\left[P(S) - P(S|G)\,P(G)\right]$ (good exercise)

$= \dfrac{1}{.47}\left[.4 - \dfrac{32}{53}\,.53\right] = \dfrac{[illegible]}{.47} = .170$

$P(M|G^c) = \dfrac{1}{P(G^c)}\left[P(M) - P(M|G)P(G)\right] = \dfrac{1}{.47}\left[.5 - \dfrac{20}{53}\,.53\right]$

$= .638$

$P(W|G^c) = \dfrac{1}{.47}\left[P(W) - P(W|G)\,P(G)\right]$

$= \dfrac{1}{.47}\left[.1 - \dfrac{1}{53}(.53)\right] = .1915$

5. B = Brown is out of town, J = Jones is out of town

$P(B) = .25$, $P(J) = .4$

(a) $P(B \cup J) = P(B) + P(J) - P(BJ)$

$= P(B) + P(J) - P(B)P(J)$, by independence,

$= .25 + .4 - (.25)(.4)$

$= \boxed{.55}$

5) (b) $P(B \mid B \cup J) = \dfrac{P(B)}{P(B)+P(J)-P(A)P(B)}$

$= \dfrac{.25}{.55} = \boxed{5/11}$

(c) $P(B \mid J) = .3$

$P(B \cup J) = P(B) + P(J) - P(BJ)$

$P(BJ) = P(B \mid J)P(J) = (.3)(.4) = .12$

$P(B \cup J) = .25 + .4 - .12 = \boxed{.53}$