

1. Consider the following crap game. Toss one fair die. Suppose the result is x . If $x = 1$ or 2 you lose. If $x = 3$ you win. If $x = 4, 5$ or 6 you continue to roll until a 1 comes up, in which case you lose, or a number with $y \geq x$ comes up, in which case you win. Find the probability that you win.

2. Suppose you roll a fair die 4 times. Find the following probabilities:

- (a) $P(\text{always } \leq 2)$.
- (b) $P(\leq 2 \text{ on first two rolls and } \geq 2 \text{ on last two.})$
- (c) $P(\text{at least one "2" .})$
- (d) $P(\text{all outcomes are the same.})$
- (e) $P(\text{exactly one "2"})$
- (f) $P(\text{exactly two "2"s.})$

3. Suppose that Akron and Bismarck are playing in the World Series, that Akron has probability p to win each game, and that the outcomes of successive games are independent.

(a) Let $g(p)$ be the probability that Akron will win the Series. It is a polynomial in p . Express g as simply as you can. Introducing $q = 1 - p$ could be helpful. Make sure that $g(1/2) = 1/2$. Then find the smallest value of p for which $g(p) \geq .8$.

(b) Suppose this time that $p = 1/2$. Find the probability that the team which wins the first game will win the Series.

4. Suppose that the r.v. X has pmf p , with

$$p(1) = .5, \quad p(2) = .3, \quad p(3) = .2.$$

Find:

- (a) The distribution function F of X .
- (b) EX and $E(e^X)$.
- (c) $E(X^2)$ and $\text{Var } X$.

5. Suppose that the r.v. X has the following distribution function F :

$$\begin{aligned} F(x) &= 0 & \text{if } x < 1.5, & & F(x) &= 1/4 & \text{if } 1.5 \leq x < 4, \\ F(x) &= 2/3 & \text{if } 4 \leq x < 6, & & F(x) &= 3/4 & \text{if } 6 \leq x < 8, \\ F(x) &= 1 & \text{if } x \geq 8. \end{aligned}$$

- (a) Sketch the graph of F .
- (b) Write down the pmf of F .
- (c) Find

$$P(1.5 < X < 5), \quad P(1.5 < X \leq 5), \quad P(1.5 \leq X < 5), \quad P(1.5 \leq X \leq 5).$$

Recommended: Problems 17 and 19 in Chapter 4.