Let W = event "you win", E_j = "j on 1'st roll"

(1)

$$P(\overline{W}) = \sum_{j=1}^{6} P(W|E_j)\,P(E_j)$$

$$P(E_j) = \frac{1}{6}, \quad 1 \le j \le 6$$

$$P(W|E_1) = P(W|E_2) = 0$$

$$P(W|E_3) = 1$$

$$P(W|E_4) = P(\text{"1" after } \{4,5,6\}) = \frac{1/6}{1/6 + \frac{1}{2}} = \frac{3}{4}$$

$$P(W|E_5) = P(\text{"1" after } \{5,6\}) = \frac{1/6}{1/6 + \frac{1}{3}} = \frac{2}{3}$$

$$P(W|E_6) = \frac{1/6}{1/6 + 1/6} = \frac{1}{2}$$

$$P(W) = \frac{1}{6}\left[1 + \frac{3}{4} + \frac{2}{3} + \frac{1}{2}\right] = \boxed{35/72} = .486$$

(5) B = Brown out of town, J = Jones out of town

$P(B) = .25$, $P(J) = .4$

(a) $P(B \cup J) = P(B) + P(J) - P(JB) = P(B) + P(J) - P(J)P(B)$

$$= .25 + .4 - (.25)(.4)$$

$$= \boxed{.55}$$

2 Roll fair die 4 times

(a) $P(\text{always} \le 2) = \left(\frac{1}{3}\right)^4$

(b) $P(\le 2$ on rolls one and two, ≥ 2 on rolls three, four$)$

$= \left(\frac{1}{3}\right)^2 \left(\frac{5}{6}\right)^2$

(c) $P(\text{at least one "2"}) = 1 - P(\text{no "2"}) = 1 - \left(\frac{5}{6}\right)^4$

(d) $P(\text{all } j\text{'s}) = \left(\frac{1}{6}\right)^4$, for $j = 1, 2, \ldots, 6$.

$P(\text{all outcomes are same.}) = 6 \frac{1}{6^4} = 6^{-3}$

(e) $P(\text{"2" on } j\text{'th roll, non "2" on other rolls})$

$= \left(5/6\right)^3 \frac{1}{6}$

$P(\text{exactly one "2"}) = \left(5/6\right)^3 \frac{1}{6} \cdot 4$

(f) $P(\text{"2" on 1'st and 2'nd rolls, non-"2" on 3'rd and 4'th})$

$= \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$

$P(\text{exactly two "2"'s}) = \binom{4}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$

$g(p) =$

<u>R3</u> $P(A \text{ wins series}) = \sum_{j=4}^{7} P(E_j)$, $\quad p = P(A$ wins a particular game

where E_j = event series ends after j games, and A wins.

$P(E_4) = p^4$, $\quad P(E_5) = p^4 q \cdot 4$, $\quad$ where $q = 1-p$

$P(E_6) = p^4 \cdot q^2 \binom{5}{2} = 10\, p^4 q^2$

$P(E_7) = p^4 \cdot q^3 \binom{6}{3} = 20\, p^4 q^3$

* $g(p) = p^4 \left[1 + 4(1-p) + 10(1-p)^2 + 20(1-p)^3 \right]$

$= (1-q)^4 \left[1 + 4q + 10q^2 + 20q^3 \right]$, $\quad q = 1-p$

smallest p s.t. $g(p) \geq .8$ is $\simeq .65$

(b) $p = 1/2$

$P(A \text{ wins series} \mid A \text{ wins 1'st game}) = \sum_{j=4}^{7} P(E_j \mid F)$

where F = event "A wins 1'st game"

$P(E_4 \mid F) = P(\text{aaa in games } 2,3,4) = 1/8$

$P(E_5 \mid F) = 3 \cdot \left(\frac{1}{2}\right)^4 = 3/16$

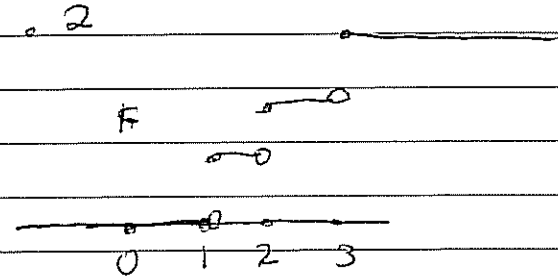
$P(E_6 \mid F) = \binom{4}{2} \cdot \left(\frac{1}{2}\right)^5 = 6/32$

$P(E_j \mid F) = \binom{5}{3} \left(\frac{1}{2}\right)^6 = 10/64$

$P(A \text{ wins series} \mid F) = \frac{42}{64} = \boxed{\frac{21}{32}}$

4 $p(1) = .5, \quad p(2) = .3, \quad p(3) = .2$

(a) $F(x) = 0, \quad \text{if } -\infty < x < 1$



$= .5, \quad \text{if } 1 \le x < 2$

$= .8, \quad \text{if } 2 \le x < 3$

$= 1, \quad \text{if } x \ge 3$

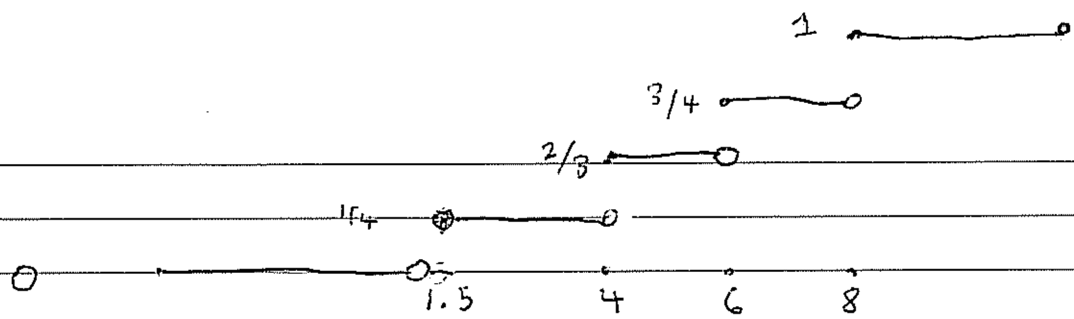
(b) $EX = 1(.5) + 2(.3) + 3(.2) = 1.7$

$E(e^X) = e(.5) + .3e^2 + .2e^3$

(c) $E(X^2) = 1^2(.5) + 2^2(.3) + 3^2(.2) = 3.5$

$\text{Var } X = E(X^2) - (EX)^2 = 3.5 - (1.7)^2 = .61$

5 (a)



F is a step function with $F(x) = 0 \ \forall x < 1.5$, $F(x) = 1 \ \forall x \ge 8$, and F takes on values $\frac{1}{4}, \frac{2}{3}, \frac{3}{4}$ on the intervals shown

(b) $p(x) =$ jump of F at x, so

$p(1.5) = .25$, $p(4) = \frac{2}{3} - \frac{1}{4} = 5/12$

$p(6) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$, $p(8) = \frac{1}{4}$,

$p(x) = 0 \ \forall$ other $x \in \mathbb{R}$.

(c) $F(x) = P(X \le x) = \sum_y p(y)$

where the sum is over all y such that $y \le x$ and $p(y) > 0$.

So in general, $P(a < X < b) = P(X < b) - P(X \le a)$

$= \sum_y p(y)$

where sum is over all y s.t. $a < y < b$ and $p(y) > 0$.

In particular: $P(1.5 < X < 5) = p(4) = 5/12$

$P(1.5 < X \leq 5) = p(4) = 5/12$

$P(1.5 \leq X < 5) = p(1.5) + p(4) = 8/12$

$P(1.5 \leq X \leq 5) = p(1.5) + p(4) = 8/12$