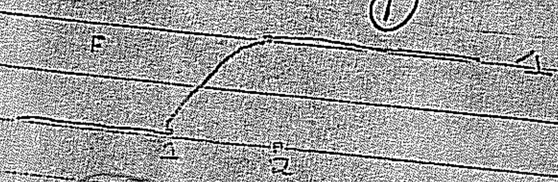
(1)

$F(x) = c \log x$ on $1 \le x \le 2$



Need $F(2) = 1$, so

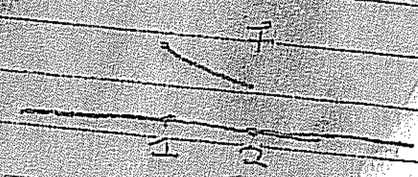
$$\boxed{c = \frac{1}{\log 2}}$$

HW 6

$f(x) = F'(x)$ $\quad F'(x) = c/x$ on $1 < x < 2$

$f(x) = \frac{1}{\log 2} \frac{1}{x}$, if $1 < x < 2$

$f(x) = 0$ for other x



(2) (a) $f(x) = 10x^{-2}, \quad x > 10$

$P(X > 25) = \int_{25}^{\infty} 10x^{-2}\,dx = -\frac{10}{x}\Big|_{25}^{\infty} = \frac{10}{25} = .4$

(c) Let $\alpha = P(X > 25) = 0.4$. Assume the trials are independent.

P(at least 3 of 6 will go ≥ 25 hours)

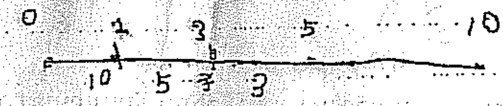
$= P(Y \geq 3)$, where $Y \sim B(6, .4)$

$= 1 - (1-\alpha)^6 - 6\alpha(1-\alpha)^5 - \binom{6}{2}\alpha^2(1-\alpha)^4$

$= 1 - (.6)^6 - (6)(.4)(.6)^5 - 15(.4)^2(.6)^4$

$= 1 - .046656 - .186624 - .31104$

$= .45568$



(3) $Y = \#$ of points on one shot

Let $f = f_Y$. Then $f(y) = P(Y = y)$ for $y = 10, 5,$ or 3, or 0

$f(3) = .2$, $f(5) = .2$, $f(10) = .1$, $f(0) = .5$

$EY = 3\cdot(.2) + 5\cdot(.2) + 10\cdot(.1) = \boxed{2.6}$

(4) $X \sim N(5, \sigma^2)$. $P(X > 9) = .2$ Find σ^2.

Let $Z = \frac{X-5}{\sigma}$. Then $P(X > 9) = P(Z > 4/\sigma)$. So

$P(Z > 4/\sigma) = .2$, $Z \sim N(0,1)$. So

$\Phi(4/\sigma) = .8$

By Table, $4/\sigma \approx .84$, $\sigma \approx 4.76$, $\boxed{\mathrm{Var}\, X = 22.7}$

(5) $X \sim N(\mu, \sigma^2)$, $\mu = 71$, $\sigma^2 = 6.25$, $\sigma = 2.5$.

Let $Z = \frac{X-71}{2.5}$. Then $Z \sim N(0,1)$

(a) $P(X > 74) = P(Z > 3/2.5) = P(Z > 1.2)$

$= 1 - \Phi(1.2) = 1 - .8849 = \boxed{.1159}$

(b) $P(X > 77 \mid X \geq 72) = P(Z > 2.4 \mid Z > .4)$

$$= \frac{P(Z > 2.4)}{P(Z > .4)} = \frac{1 - \Phi(2.4)}{1 - \Phi(.4)} = \frac{1 - .9918}{1 - .6554}$$

$$= \frac{.0082}{.3446} = \boxed{.0238}$$

(6) Let X = # of unacceptables in lot of 150.

Then $X \sim B(150, .05)$.

$EX = np = 7.5$

$\mathrm{Var}\, X = npq = (7.5)(.95) = 7.125$

Using the normal approx., $Z = \dfrac{X - 7.5}{(7.125)^{1/2}} \approx N(0,1)$

$(X \le 10) = P(X \le 10.5) = P\left(Z \le \dfrac{3}{(7.125)^{1/2}}\right) \approx \Phi(1.12) = \boxed{.8686}$

Poisson approx .8622, Exact probab. .8678

(7) $X \sim \exp(1/6)$. Then $P(X > x) = e^{-x/6}$, for $x \ge 0$

(a) $P(X > 2) = e^{-2 \cdot \frac{1}{6}} = e^{-1/3} = \boxed{.717}$

(b) $P(X > 10 \mid X > 9) = P(X > 1) = e^{-1/6} = .846$,

using memoryless property.

$$F(t) = 1 - e^{-\int_0^t \lambda(s)\,ds}$$, Here X = the lifetime (5)

and $F = F_X$, $t > 0$. Suppose $\lambda(t) = t^3$. Then

$\int_0^t s^3\,ds = \frac{1}{4}t^4$

$F(t) = 1 - e^{-\frac{1}{4}t^4}$

(a) $P(X > 2) = 1 - F(2) = \boxed{e^{-4}} = .0183$

(b) $P(.4 < X < 1.4) = F(1.4) - F(.4) = e^{-\frac{1}{4}(.4)^4} - e^{-\frac{1}{4}(1.4)^4}$

$= e^{-.0064} - e^{-.9604} = .994 - .383 = \boxed{.611}$

(c) $P(X > 2 \mid X > 1) = \dfrac{P(X > 2)}{P(X > 1)} = \dfrac{e^{-4}}{e^{-1/4}} = \boxed{e^{-15/4} = .0235}$

d.f. of X is $F(t) = 1 - e^{-\frac{1}{4}t^4}$, $0 < t < \infty$

$= 0$, $-\infty < t \leq 0$

p.d.f. of X is $f(t) = t^3 e^{-\frac{1}{4}t^4}$, $t > 0$

$= 0$, $t < 0$

Comparison Take $s = t = 1$. Then

$P(X > 2 \mid X > 1) = e^{-\frac{15}{4}} = .0235$

$P(X > 1) = e^{-1/4} = .779$

$P(X > 2 \mid X \geq 1) << P(X > 1)$

Question If $\lambda(t)$ is an strictly increasing function on $(0, \infty)$

do we have

$P(X > t+s \mid X > t) < P(X > s)$

For every $t, s > 0$?