Math 493. Homework 8 Due November 13, 2009

Unless otherwise stated, problem citations are from Problems for Chapter 6, pp.287-291.

1. Suppose that X_1, X_2, X_3 form a random sample of size 3 from a uniform distribution on $[0,1]$. Let

$$Y = X_1 + X_2 + X_3, \quad Z = \frac{1}{3}(X_1 + X_2 + X_3),$$

and let g and h denote the pdf's of Y and Z, respectively.

(a) g requires several different pieces for its specification. One piece is

$$g(x) = \frac{1}{2}(3-x)^2, \quad \text{if } 2 < x < 3.$$

Verify this for yourself, and find the rest of g.

(b) Express h in terms of g. Then sketch, on the same set of axes, the graphs of h, the p.d.f. of $\frac{1}{2}(X_1 + X_2)$ and the pdf of X_1. What do you think the p.d.f of $\frac{1}{n}\sum_{i=1}^{n} X_i$ will look like for large n, when $X_1, ..., X_n$ form a random sample of size n from $U(0,1)$?

2. Problem 40.

3. Let T be the triangle with vertices at $(0,0), (1,0), (1,2)$. Suppose that the jointly continuous pair of r.v.'s (X, Y) has p.d.f $f(x,y) = \frac{5}{2}x^2y$, for $(x,y) \in T$, and $f(x,y) = 0$ for other (x,y).

(a) For which real numbers y does the conditional distribution of X given $Y = y$ exist? For those y, compute the p.d.f of X given $Y = y$.

(b) Repeat (a), with X and Y interchanged.

(c) Find $P(X < 3/4 | Y = 1)$.

4. Problem 47, but change $(1/4, 3/4)$ to (a, b), where 0 □ $a < b$ □ 1.

5. Suppose that X_1 and X_2 form a random sample of size 2 from a distribution whose p.d.f. is $f(x) = 2x$ for $0 < x < 1$ and $f(x) = 0$ for other x. Find:

(a) The d.f. F of X_1.

(b) $P(\max\{X_1, X_2\} \square 2/3)$.

(c) $P(\min\{X_1, X_2\} \square 2/3)$.

(d) $P(\min\{X_1, X_2\} \geq 1/2 \text{ and } \max\{X_1, X_2\} \square 2/3)$.

Recommended problems: 14, 29,31, 36, 42, 44, 48.