Math 493. Homework 9 Due November 20, 2009

1. Suppose that (X, Y) has a continuous joint distribution with pdf $f(x, y) = \frac{5}{2}x^2y$ for (x, y) in the triangle T with vertices $(0,0), (1,0), (1,2)$ and $f(x, y) = 0$ outside T. Define new r.v.'s by

$$U = 2X + Y, \quad V = X + 3Y.$$

Let k be the joint p.d.f. of U and V.

(a) Sketch the set $S = \{(u, v) : k(u, v) > 0\}$. And what is the area of S?

(b) Calculate $k(4/3, 3/2)$.

2. Problem 55, p.290. You may use the Jacobian calculation I did in class.

3. Problem 56(b), p.290.

4. (a) Let f and g be nonnegative functions on $(-\infty, \infty)$ which vanish on $(-\infty, 0)$. Show, for yourself, that the convolution $f * g$ vanishes when $x < 0$, and that

$$f * g(x) = \int_0^x f(y)g(x - y)\, dy = \int_0^x f(x - y)g(y)\, dy,$$

when $0 < x < \infty$.

(b) Let X and Y be independent exponential r.v.'s with respective parameters λ_1 and λ_2. Calculate the p.d.f. h of $X + Y$ when $\lambda_1 \neq \lambda_2$.

(c) With X and Y as in (b), suppose that $\lambda_1 = \lambda_2 = \lambda$. Use convolutions to find the pdf h of $X + Y$. The distribution of $X + Y$ is among those found in Section 6.3. What is it called?

5. Problem 39, p.375.

Recommended problems: On pp. 316-319: 27, 52. On pp. 373-379: 6, 30, 31, 33, 36, 37, 38, 41, 45.