Math 132
Fall 2007 Exam II

Integral Formula:

$$\int \sec(t)^3\, dt = \frac{1}{2}\sec(t)\tan(t) + \frac{1}{2}\ln(|\sec(t)+\tan(t)|) + C$$

1. Suppose that $f(x) = 2^{\sqrt{x}}$. Calculate $D(f)(9)$, the derivative of $f(x)$ evaluated at x = 9.)

a) $\frac{2}{3}\ln(2)$ b) $\frac{2}{9}\ln(2)$ $\boxed{c)}$ $\frac{4}{3}\ln(2)$ d) $\frac{4}{9}\ln(2)$ e) $\frac{4}{27}\ln(2)$

f) $\frac{2}{3\ln(2)}$ g) $\frac{2}{9\ln(2)}$ h) $\frac{4}{3\ln(2)}$ i) $\frac{4}{9\ln(20)}$ j) $\frac{4}{27\ln(2)}$

$f(x) = e^{\sqrt{x}\ln 2}$

$f'(x) = \frac{1}{2\sqrt{x}}\ln 2 \cdot e^{\sqrt{x}\ln 2}$

$f'(x) = \frac{\ln 2}{2\sqrt{x}} \cdot 2^{\sqrt{x}}$

$f'(9) = \frac{\ln 2}{2\cdot 3} \cdot 2^3 = \frac{4\ln 2}{3}$

2. Calculate $\int_1^3 \log_3(x)\,dx$.

a) $2-\frac{1}{\ln(3)}$ b) $2+\frac{1}{\ln(3)}$ c) $2-\ln(3)$ d) $2+\ln(3)$ e) $2\ln(3)-1$

f) $2\ln(3)+1$ $\boxed{g)}$ $3-\frac{2}{\ln(3)}$ h) $3+\frac{2}{\ln(3)}$ i) $1-\frac{2}{\ln(3)}$ j) $1+\frac{2}{\ln(3)}$

$$\int_1^3 \log_3 x\,dx = \int_1^3 \frac{\ln x}{\ln 3}\,dx = \frac{1}{\ln 3}\int_1^3 \ln x\,dx$$

$$= \frac{1}{\ln 3}\left(x\ln x - x\right)\Big|_1^3 = \frac{1}{\ln 3}\left(3\ln 3 - 3 - (1\cdot 0 - 1)\right)$$

$$= \frac{3\ln 3 - 2}{\ln 3} = 3 - \frac{2}{\ln 3}$$

3. Suppose that $f(x) = x^{\sqrt{x}}$. Calculate D(f)(4). (D(f)(4) is the derivative of f(x) evaluated at x = 4.)

a) $1+\ln(2)$ b) $2(2+\ln(2))$ c) $4(2+\ln(2))$ d) $2(1+\ln(2))$ e) $8+\ln(2)$

f) $2+\ln(2)$ g) $4+\ln(2)$ h) $4(1+\ln(2))$ i) $2(4+\ln(2))$ j) $8(1+\ln(2))$

$$f(x) = e^{\sqrt{x}\ln x}$$

$$f'(x) = (\sqrt{x}\ln x)' \cdot e^{\sqrt{x}\ln x}$$

$$f'(x) = \left(\frac{1}{2\sqrt{x}} \cdot \ln x + \frac{\sqrt{x}}{x}\right) \cdot x^{\sqrt{x}}$$

$$f'(4) = \left(\frac{1}{2 \cdot 2} \cdot \ln 4 + \frac{2}{4}\right) \cdot 4^2$$

$$= \left(\frac{\ln 2}{2} + \frac{1}{2}\right) \cdot 16$$

$$= 8(\ln 2 + 1)$$

4. A radioactive substance has mass 120g at time $t=4$ and mass 90g at time $t=6$. What is the mass at $t=12$?

a) $\frac{1979}{32}$	b) $\frac{1985}{32}$	c) $\frac{1991}{32}$	d) $\frac{1997}{32}$	e) $\frac{1203}{21}$
f) $\frac{1209}{32}$	g) $\frac{1215}{32}$	h) $\frac{1221}{32}$	i) $\frac{1227}{32}$	j) $\frac{1233}{32}$

Let $w = t-4$

Mass	w
120	0
90	2
?	8

$$y = A_0 e^{-Kw}$$

$$y = 120\, e^{-Kw}$$

$$90 = 120\, e^{-2K}$$

$$\frac{3}{4} = e^{-2K}$$

$$\ln\left(\frac{3}{4}\right) = -2K$$

$$K = -\frac{1}{2}\ln\left(\frac{3}{4}\right)$$

$$y = 120\, e^{\frac{1}{2}\ln\left(\frac{3}{4}\right)\cdot w}$$

$$y = 120\, e^{\frac{1}{2}\ln\left(\frac{3}{4}\right)\cdot 8}$$

$$y = 120\, e^{4\ln\left(\frac{3}{4}\right)}$$

$$= 120\left(\frac{3}{4}\right)^4 = 120\cdot\frac{81}{256} = \frac{8\cdot 15\cdot 81}{256} = \frac{15\cdot 81}{32} = \frac{1215}{32}$$

$$\begin{array}{r} 81 \\ \times\ 15 \\ \hline 405 \\ 810 \\ \hline \end{array}$$

5. **The mass of a microbe colony splashing about in a nutrient broth triples every 12 hours. What is the colony's doubling time?**

a) 8 b) $8\ln(2)$ c) $8\ln(3)$ d) $\dfrac{8\ln(2)}{\ln(3)}$ e) $\dfrac{12\ln(2)}{\ln(3)}$

f) $12\ln\left(\dfrac{3}{2}\right)$ g) $\dfrac{3\ln(12)}{2}$ h) $3\ln(2)$ i) $2\ln(3)$ j) $\dfrac{2\ln(12)}{3}$

$$y = A_0 e^{kt}$$

$$3A_0 = A_0 e^{k \cdot 12}$$

$$\frac{\ln 3}{12} = k$$

$$y = A_0 e^{\frac{\ln 3}{12} t}$$

$$2A_0 = A_0 e^{\ln 3 \cdot \frac{t}{12}}$$

$$\ln 2 = \ln 3 \cdot \frac{t}{12}$$

$$t = 12 \frac{\ln 2}{\ln 3}$$

6. Suppose that $u(t)$ is the unique solution of the initial value problem

$$\frac{d}{dt}u(t) = M - 5\,u(t),\ u(0) = 1 \quad \textbf{where } M \textbf{ is a constant.}$$

If $\lim_{t \to \infty} u(t) = 12$ **then what is** M**?**

a) 3	b) 4	c) 5	d) 10	e) 12
f) 15	g) 20	h) 30	$\boxed{\text{i)}}$ 60	j) 120

$$\frac{du}{dt} = M - 5u$$

$$\frac{du}{M-5u} = dt$$

$$-\frac{1}{5}\ln M - 5u = t + C$$

$$M - 5u = Ae^{-\frac{t}{5}}$$

$$M = 5u + Ae^{-\frac{t}{5}}$$

$u(0)=1 \Rightarrow M = 5 + A$

$u(\infty)=12 \Rightarrow M = 60$

7. **Suppose that $f(x) = \operatorname{arcsec}(5x)$. Calculate $D(f)\left(-\frac{1}{3}\right)$. (The derivative of $f(x)$ at $x = -\frac{1}{3}$).**

a) $-\frac{9}{4}$	b) $-\frac{5}{4}$	c) $-\frac{4}{3}$	d) $-\frac{5}{3}$	e) $-\frac{9}{20}$
$\boxed{\text{f)}}$ $\frac{9}{4}$	g) $\frac{5}{4}$	h) $\frac{4}{3}$	i) $\frac{5}{3}$	j) $\frac{9}{20}$

$$f'(x) = \frac{1}{|5x|\sqrt{(5x)^2-1}} \cdot 5$$

$$f'\left(-\frac{1}{3}\right) = \frac{5}{\frac{5}{3}\sqrt{(\frac{5}{3})^2-1}} = \frac{3}{\sqrt{\frac{25}{9}-1}} = \frac{3}{\sqrt{\frac{16}{9}}} = \frac{3}{\frac{4}{3}} = \frac{9}{4}$$

$f(x) = 120 \arctan(\sqrt{x})$

8. Suppose that ~~f(x) = arctan($\sqrt{x}$)~~. **What is D(f)(4)? (The derivative of f(x) at $x = 4$).**

a) 2	b) 3	c) 4	d) 5	e) 6
f) 8	g) 10	h) 12	i) 15	j) 20

$$f'(x) = \frac{120}{(\sqrt{x})^2+1} \cdot \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{120}{4+1} \cdot \frac{1}{2 \cdot 2} = \frac{120}{20} = 6.$$

9. Calculate $\displaystyle\int_0^{\frac{1}{2}} x\, e^{(2x)}\, dx.$

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) 1 e) 2

f) $\frac{e}{4}$ g) $\frac{e}{2}$ h) $\frac{3e}{4}$ i) e j) 2 e

$u = x \qquad v = \frac{1}{2} e^{2x}$

$du = dx \qquad dv = e^{2x} dx$

$= \frac{1}{2} x e^{2x} \Big|_0^{\frac{1}{2}} - \frac{1}{2}\int_0^{\frac{1}{2}} e^{2x}\, dx$

$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \Big|_0^{\frac{1}{2}}$

$= \left(\frac{1}{2}\left(\frac{1}{2}\right) e - \frac{1}{4} e\right) - \left(0 - \frac{1}{4}\right)$

$= \frac{1}{4}$

10. Calculate $\int_0^{\pi} x^2 \sin(x)\, dx$.

a) π^2 b) $\pi^2 - 1$ c) $\pi^2 - 2$ d) $\pi^2 - 3$ e) $\pi^2 - 4$
f) $\pi^2 + 5$ g) $\pi^2 + 4$ h) $\pi^2 + 3$ i) $\pi^2 + 2$ j) $\pi^2 + 1$

$u = x^2$ $\quad v = -\cos x$

$du = 2x\,dx$ $\quad dv = \sin x\, dx$

$= -x^2 \cos x \Big|_0^{\pi} + \int_0^{\pi} 2x \cos x\, dx$

$u = x$ $\quad v = \sin x$

$du = dx$ $\quad dv = \cos x\, dx$

$= -x^2 \cos x \Big|_0^{\pi} + 2\left(x \sin x \Big|_0^{\pi} - \int_0^{\pi} \sin x\, dx\right)$

$= -x^2 \cos x + 2x \sin x + 2\cos x \Big|_0^{\pi}$

$= -\pi^2 \cdot (-1) + 2\pi \cdot 0 + 2(-1) - (0 + 0 + 2)$

$= \pi^2 - 4$

11. **Calculate** $25\int_1^e x^4 \ln(x)\,dx$.

a) e^5-1 b) $2e^5-1$ c) $3e^5-1$ d) $4e^5-1$ e) $5e^5-1$

f) e^5+1 g) $2e^5+1$ h) $3e^5+1$ i) $4e^5+1$ j) $5e^5+1$

$u = \ln x \qquad v = \frac{x^5}{5}$

$du = \frac{1}{x}\,dx \qquad dv = x^4\,dx$

$= 25\left(\frac{x^5}{5}\ln x \Big|_1^e - \int_1^e \frac{x^4}{5}\,dx \right)$

$= 25\left(\frac{x^5}{5}\ln x - \frac{x^5}{25}\Big|_1^e \right)$

$= 5e^5 - e^5 - (-1)$

$= 4e^5 + 1$

12. Calculate $\displaystyle\int_1^2 \frac{5x+2}{x^2+x}\,dx$.

a) $\ln(2)$ b) $\ln(3)$ c) $2\ln(2)$ d) $2\ln(3)$ e) $3\ln(2)$

f) $\ln(6)$ g) $\ln\left(\frac{2}{3}\right)$ h) $\ln\left(\frac{3}{2}\right)$ i) $\ln\left(\frac{9}{2}\right)$ j) $\ln\left(\frac{27}{2}\right)$

$$\frac{5x+2}{x^2+x} = \frac{A}{x} + \frac{B}{x+1}$$

$$5x+2 = A(x+1) + Bx$$

$$x=0 \Rightarrow 2 = A$$

$$x=-1 \Rightarrow -3 = -B$$

$$\Rightarrow 3 = B$$

$$= \int_1^2 \frac{2}{x} + \frac{3}{x+1}\,dx$$

$$= 2\ln x + 3\ln(x+1)\Big|_1^2$$

$$= 2\ln 2 + 3\ln 3 - (2\ln 1 + 3\ln 2)$$

$$= 3\ln 3 - \ln 2 = \ln\left(\frac{27}{2}\right)$$

13. **Find an ordered triple (α, β, γ) of positive integers α, β, γ such that**

$$\int_1^2 \frac{x^2+3x-4}{x(x+2)^2}\,dx = \alpha \ln(\beta) - \beta \ln(\alpha) + \frac{1}{\gamma}.$$

a) 2, 3, 4 **b)** (3, 2, 4) **c)** (4, 3, 2) **d)** (4, 2, 3) **e)** 3, 4, 2
f) (3, 2, 3) **g)** (2, 3, 2) **h)** (4, 2, 2) **i)** (4, 3, 3) **j)** (3, 2, 2)

$$\frac{x^2+3x-4}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$x^2+3x-4 = A(x+2)^2 + Bx(x+2) + Cx$$

$x = 0 \Rightarrow \quad -4 = A \cdot 4$

$-1 = A$

$x = -2 \Rightarrow \quad -6 = -2C$

$3 = C$

x^2 coefficient $\Rightarrow \quad 1 = A + B$

$1 = -1 + B$

$2 = B$

$$= \int_1^2 -\frac{1}{x} + \frac{2}{x+2} + \frac{3}{(x+2)^2}\,dx$$

$$= -\ln x + 2\ln(x+2) - \frac{3}{x+2}\Big|_1^2$$

$$= -\ln 2 + 2\ln 4 - \frac{3}{4} - \left(0 + 2\ln 3 - \frac{3}{3}\right)$$

$$= 3\ln 2 - 2\ln 3 + \frac{1}{4}$$

14. Calculate $\displaystyle\int_0^{\pi} \sin(x)^3 \cos(x)^2\,dx.$

a) 1/15	b) 2/15	c) 1/5	d) 4/15	e) 1/3
f) 2/5	g) 7/15	h) 8/15	i) 3/5	j) 2/3

$u = \cos x$

$du = -\sin x \, dx$

$$= -\int_1^{-1} (1-u^2)\,u^2\,du$$

$$= \int_{-1}^{1} u^2 - u^4 \,du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} \Big|_{-1}^{1}$$

$$= \frac{1}{3} - \frac{1}{5} - \left(-\frac{1}{3} + \frac{1}{5}\right) = \frac{2}{3} - \frac{2}{5} = \frac{10}{15} - \frac{6}{15} = \frac{4}{15}$$

15. Use the reduction formula

$$\int_0^1 x^n e^{(-x^2)}\,dx = \frac{(n-1)\int_0^1 x^{(n-2)} e^{(-x^2)}\,dx}{2} - \frac{1}{2e}$$

and the approximation $\int_0^1 e^{(-x^2)}\,dx \simeq \frac{3}{4}$ to approximate $\int_0^1 x^4 e^{(-x^2)}\,dx$.

a) $\frac{1}{2}-\frac{1}{4e}$ b) $\frac{1}{2}-\frac{3}{4e}$ c) $\frac{3}{4}-\frac{1}{2e}$ d) $\frac{3}{4}-\frac{3}{2e}$ e) $\frac{9}{16}-\frac{3}{4e}$

f) $\frac{9}{16}-\frac{5}{4e}$ g) $\frac{5}{8}-\frac{5}{4e}$ h) $\frac{5}{8}-\frac{3}{4e}$ i) $\frac{3}{4}-\frac{1}{e}$ j) $\frac{9}{16}-\frac{1}{e}$

$$\int_0^1 x^4 e^{-x^2} = \frac{(4-1)}{2}\int_0^1 x^2 e^{-x^2}\,dx - \frac{1}{2e}$$

$$= \frac{3}{2}\left(\frac{2-1}{2}\int_0^1 e^{-x^2}\,dx - \frac{1}{2e}\right) - \frac{1}{2e}$$

$$\approx \frac{3}{2}\left(\frac{1}{2}\cdot\frac{3}{4} - \frac{1}{2e}\right) - \frac{1}{2e}$$

$$\approx \left(\frac{9}{16} - \frac{3}{4e} - \frac{1}{2e}\right)$$

$$\approx \frac{9}{16} - \frac{5}{4e}$$

16. Calculate $\displaystyle\int_{\frac{1}{\sqrt{2}}}^{1} \sqrt{1-x^2}\,dx.$

a) $\dfrac{\pi-2}{8}$ b) $\dfrac{\pi-2}{4}$ c) $\dfrac{\pi-2}{2}$ d) $\dfrac{\pi-1}{8}$ e) $\dfrac{\pi-1}{4}$

f) $\dfrac{\pi-1}{2}$ g) $\dfrac{4-\pi}{8}$ h) $\dfrac{4-\pi}{4}$ i) $\dfrac{4-\pi}{2}$ j) $\dfrac{8-\pi}{8}$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} |\cos\theta|\cos\theta\, d\theta$ $\qquad \cos 2\theta = 2\cos^2\theta - 1$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos 2\theta + 1}{2}\, d\theta$

$= \frac{1}{4}\sin 2\theta + \frac{1}{2}\theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \frac{1}{4}(0) + \frac{\pi}{4} - \left(\frac{1}{4} + \frac{\pi}{8}\right)$

$= \frac{\pi}{8} - \frac{1}{4}$

17. **Calculate** $\displaystyle\int_0^1 \frac{x^2}{\sqrt{1+x^2}}\,dx.$

a) $1-\dfrac{\ln(1+\sqrt{2})}{2}$

b) $1+\dfrac{\ln(1+\sqrt{2})}{2}$

c) $\sqrt{2}+\ln(1+\sqrt{2})$

d) $\sqrt{2}-\ln(1+\sqrt{2})$

e) $1-\ln(1+\sqrt{2})$

f) $1+\ln(1+\sqrt{2})$

g) $\dfrac{1}{2}-\ln\left(1+\dfrac{1}{\sqrt{2}}\right)$

h) $\dfrac{1}{2}+\ln\left(1+\dfrac{1}{\sqrt{2}}\right)$

i) $\dfrac{\sqrt{2}}{2}-\dfrac{\ln(1+\sqrt{2})}{2}$

j) $\dfrac{\sqrt{2}}{2}+\dfrac{\ln(1+\sqrt{2})}{2}$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$= \int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{\sec\theta}\cdot \sec^2\theta\, d\theta$

$= \int_0^{\frac{\pi}{4}} \tan^2\theta \sec\theta\, d\theta$

$= \int_0^{\frac{\pi}{4}} (\sec^2\theta - 1)\sec\theta\, d\theta$

$= \int_0^{\frac{\pi}{4}} \sec^3\theta - \sec\theta\, d\theta$

$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| - \ln|\sec\theta+\tan\theta| \Big|_0^{\frac{\pi}{4}}$

$= \frac{1}{2}(\sqrt{2}) - \frac{1}{2}\ln(\sqrt{2}+1)$

$-(0 - \frac{1}{2}\ln|1+0|)$

$= \frac{\sqrt{2}}{2} - \frac{\ln(\sqrt{2}+1)}{2}$

18. **Evaluate** $\displaystyle\int_0^1 \frac{4x^2+2x+4}{(x^2+1)^2}\,dx.$

a) $2\pi - 1$ b) $2\pi + 1$ c) $\pi - \frac{1}{4}$ d) $\pi - \frac{1}{2}$ e) $\pi - 1$

f) π g) $\pi + \frac{1}{4}$ h) $\pi + \frac{1}{2}$ i) $\pi + 1$ j) 2π

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$= \int_0^{\frac{\pi}{4}} \frac{4\tan^2\theta + 2\tan\theta + 4}{\sec^4\theta} \cdot \sec^2\theta\, d\theta$

$= \int_0^{\frac{\pi}{4}} 4\frac{\tan^2\theta}{\sec^2\theta} + 2\frac{\tan\theta}{\sec^2\theta} + \frac{4}{\sec^2\theta}\, d\theta$

$= \int_0^{\frac{\pi}{4}} 4\sin^2\theta + 2\sin\theta\cos\theta + 4\cos^2\theta\, d\theta$

$= \int_0^{\frac{\pi}{4}} 4 + \sin 2\theta\, d\theta$

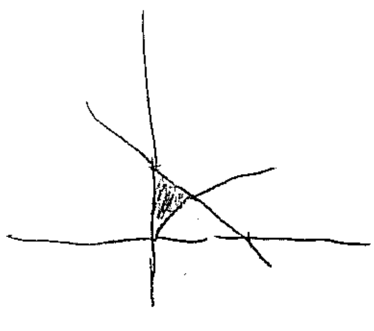
$= 4\theta - \frac{1}{2}\cos 2\theta \Big|_0^{\frac{\pi}{4}}$

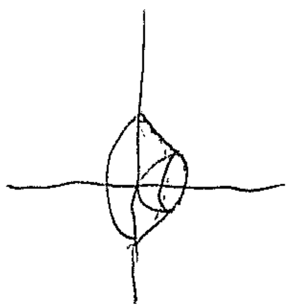
$= \pi - 0 - (0 - \frac{1}{2}) = \pi + \frac{1}{2}$

19. **The region in the first quadrant bounded above by $y = 2 - x$ and below by $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the resulting solid of revolution?**

a) 2π b) $\frac{11\pi}{6}$ c) $\frac{5\pi}{3}$ d) $\frac{3\pi}{2}$ e) $\frac{4\pi}{3}$

f) $\frac{5\pi}{2}$ g) $\frac{8\pi}{3}$ h) $\frac{9\pi}{4}$ i) $\frac{7\pi}{4}$ j) $\frac{5\pi}{4}$





Intersection:

$2 - x = \sqrt{x}$

$x + \sqrt{x} - 2 = 0$

$(\sqrt{x} + 2)(\sqrt{x} - 1) = 0$

$\sqrt{x} = 1$

$x = 1$

Disk

$$A = \int_0^1 \pi\left((2-x)^2 - (\sqrt{x})^2\right) dx$$

$$= \pi \int_0^1 (4 - 4x + x^2 - x)\, dx$$

$$= \pi \int_0^1 4 - 5x + x^2\, dx$$

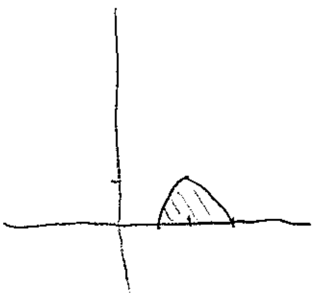
$$= \pi\left(4x - \frac{5}{2}x^2 + \frac{x^3}{3}\right)\Big|_0^1$$

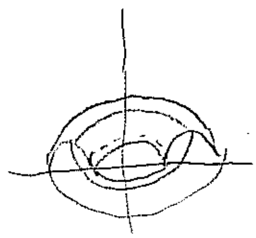
$$= \pi\left(4 - \frac{5}{2} + \frac{1}{3}\right) = \pi\left(\frac{24}{6} - \frac{15}{6} + \frac{2}{6}\right) = \pi\left(\frac{11}{6}\right)$$

20. **The region above the x-axis and under the parabola $y = 1-(x-2)^2$, $1 < x \le 3$ is rotated about the y-axis. What is the volume of the resulting solid of revolution?**

a) $\frac{5\pi}{3}$ b) $\frac{8\pi}{3}$ c) $\frac{10\pi}{3}$ d) 4π e) $\frac{9\pi}{2}$

f) 5π g) $\frac{16\pi}{3}$ h) $\frac{21\pi}{4}$ i) 6π j) $\frac{20\pi}{3}$





Shell

$A = \int_1^3 2\pi x\left(1-(x-2)^2\right)dx$

$= 2\pi\int_1^3 x\left(1-x^2+4x-4\right)dx$

$= 2\pi\int_1^3 x - x^3+4x^2-4x\,dx$

$= 2\pi\int_1^3 -x^3+4x^2-3x\,dx$

$= 2\pi\left(-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3}{2}x^2\Big|_1^3\right)$

$= 2\pi\left(-\frac{81}{4}+36-\frac{27}{2}-\left(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2}\right)\right)$

$= 2\pi\left(-\frac{135}{4}+36+\frac{1}{4}-\frac{4}{3}+\frac{3}{2}\right)$

$= 2\pi\left(-\frac{134}{4}+36-\frac{4}{3}+\frac{3}{2}\right)$

$= 2\pi\left(-\frac{67}{2}+36-\frac{4}{3}+\frac{3}{2}\right)$

$= \pi\left(5-\frac{8}{3}+3\right) = \pi\left(8-\frac{8}{3}\right)$

$= \pi\left(\frac{16}{3}\right)$