Math 132
Fall 2007 Final Exam

1. Calculate $\displaystyle\int_0^{\frac{\pi}{2}} \cos(x)\sin(x)^3\,dx.$

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$ e) $\frac{1}{5}$

f) $\frac{2}{3}$ g) $\frac{3}{4}$ h) $\frac{3}{2}$ i) $\frac{4}{3}$ j) $\frac{1}{6}$

$u = \sin x$
$du = \cos x\,dx$

$= \int_0^1 u^3\,du$

$= \frac{1}{4} u^4\Big|_0^1 = \boxed{\frac{1}{4}}$

2. Let $F(x) = \int_x^2 \frac{5+t^4}{\sqrt{1+t^3}}\,dt.$ Calculate the derivative $D(F)(2)$ of F at 2.

a) 4	b) 5	c) 6	d) 7	e) 8
f) –4	g) –5	h) –6	i) –7	j) –8

$$F(x) = -\int_2^x \frac{5+t^4}{\sqrt{1+t^3}}\,dt$$

$$F'(x) = -\frac{5+x^4}{\sqrt{1+x^3}}$$

$$F'(2) = -\frac{5+16}{\sqrt{1+8}} = -\frac{21}{\sqrt{9}} = -\frac{21}{3} = \boxed{-7}$$

3. Calculate $\displaystyle\int_0^1 \frac{x}{(x+1)(x+2)}\,dx.$

a) $\ln\left(\frac{9}{8}\right)$ b) $\ln\left(\frac{7}{6}\right)$ c) $\ln\left(\frac{5}{4}\right)$ d) $\ln\left(\frac{4}{3}\right)$ e) $\ln\left(\frac{3}{2}\right)$

f) $\ln\left(\frac{9}{5}\right)$ g) $\ln\left(\frac{8}{3}\right)$ h) $\ln\left(\frac{9}{4}\right)$ i) $\ln\left(\frac{16}{3}\right)$ j) $\ln\left(\frac{16}{9}\right)$

$$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$x = A(x+2) + B(x+1)$$

$$x = -2 \Rightarrow \quad -2 = B(-1)$$
$$2 = B$$

$$x = -1 \Rightarrow \quad -1 = A(1)$$
$$-1 = A$$

$$\therefore \int_0^1 \frac{2}{x+2} - \frac{1}{x+1}\,dx$$

$$= 2\ln(x+2) - \ln(x+1)\Big|_0^1$$

$$= 2\ln(3) - \ln(2) - (2\ln(2) - \ln(1))$$

$$= \ln(9) - \ln(8) = \boxed{\ln\left(\frac{9}{8}\right)}$$

4. Calculate $\displaystyle\int_0^1 \frac{8x^2+2x+6}{(1+x)(1+x^2)}\,dx.$

a) $\frac{1}{4}\ln(2)$ b) $\frac{1}{2}\ln(2)$ c) $\ln(2)$ d) $2\ln(2)$ e) $3\ln(2)$

f) $4\ln(2)$ g) $5\ln(2)$ h) $6\ln(2)$ i) $7\ln(2)$ j) $8\ln(2)$

$$\frac{8x^2+2x+6}{(x+1)(x^2+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$$

$$8x^2+2x+6 = (Ax+B)(x+1) + C(x^2+1)$$

$x=-1 \Rightarrow 8-2+6 = C(2)$

$12 = 2C$

$6 = C$

$$8x^2+2x+6 = (A+C)x^2 + (A+B)x + (B+C)$$
$$= (A+6)x^2 + (A+B)x + (B+6)$$

$A=2$ $B=0$

$$\int_0^1 \frac{2x}{x^2+1} + \frac{6}{x+1}\,dx = \int_0^1 \frac{2x}{x^2+1}\,dx + 6\int_0^1 \frac{1}{x+1}\,dx$$

$u=x^2+1$

$du=dx$

$$= \int_1^2 \frac{1}{u}\,du + 6\ln(x+1)\Big|_0^1$$

$$= \ln(u)\Big|_1^2 + 6\ln(x+1)\Big|_0^1$$

$$= \ln 2 - \ln 1 + 6(\ln 2 - \ln 1)$$

$$= \boxed{7\ln 2}$$

5. **Calculate** $\int_1^e x^2 \ln(x)\, dx$.

a) $\frac{1}{3}e^3$ b) $\frac{1}{3}(2e^3-1)$ c) $\frac{1}{3}(e^3-2)$ d) $\frac{2}{3}(e^3-1)$ e) $\frac{1}{3}(2e^3+1)$

f) $\frac{1}{3}(e^3+2)$ g) $\frac{2}{3}(e^3+1)$ h) $\frac{1}{9}(2e^3+1)$ i) $\frac{1}{9}(e^3+2)$ j) $\frac{2}{9}(e^3+1)$

$u = \ln(x)$ $v = \frac{1}{3}x^3$

$du = \frac{1}{x}dx$ $dv = x^2 dx$

$= \ln(x) \cdot \frac{1}{3}x^3 \Big|_1^e - \int_1^e \frac{1}{3}x^3 \cdot \frac{1}{x}\, dx$

$= \ln(e) \cdot \frac{1}{3}e^3 - \ln(1) \cdot \frac{1}{3} \cdot 1^3 - \left(\int_1^e \frac{x^2}{3}\, dx \right)$

$= \frac{e^3}{3} - \frac{1}{9}x^3 \Big|_1^e$

$= \frac{e^3}{3} - \frac{1}{9}e^3 + \frac{1}{9}$

$= \frac{1}{9}(3e^3 - e^3 + 1)$

$= \boxed{\frac{1}{9}(2e^3 + 1)}$

6. **What is the derivative of** $x^{\left(\frac{1}{x}\right)}$ **with respect to** x **at** $x=\frac{1}{2}$**?**

a) $-\ln(2)$ b) $-\frac{1}{2}\ln(2)$ c) $1-\ln(2)$ d) $1-\frac{1}{2}\ln(2)$ e) $\ln(2)$

f) $\frac{1}{2}\ln(2)$ g) $1+\ln(2)$ h) $1+\frac{1}{2}\ln(2)$ i) $\frac{1}{4}\ln(2)$ j) $\frac{1}{4}$

$$\left(x^{\frac{1}{x}}\right)'\left(\tfrac{1}{2}\right) = \left(e^{\frac{1}{x}\ln x}\right)'\left(\tfrac{1}{2}\right) = \left(-\frac{1}{x^2}\ln x + \frac{1}{x^2}\right)\left(e^{\frac{1}{x}\ln x}\right)\left(\tfrac{1}{2}\right)$$

$$= \left(-\frac{1}{\frac{1}{4}}\ln\left(\tfrac{1}{2}\right) + \frac{1}{\frac{1}{4}}\right)\left(\tfrac{1}{2}\right)^2$$

$$= (4\ln 2 + 4)\cdot\frac{1}{4}$$

$$= \boxed{\ln 2 + 1}$$

7. If $y(0)=0$ and $\frac{dy}{dx} = \cos(x)\sqrt{1-y^2}$, then what is $y(x)$?

a) $\sin(\pi\cos(x))$ b) $\sin(\sin(x))$ c) $\cos\left(\frac{\pi\cos(x)}{2}\right)$ d) $\cos(\sin(x))$ -1 e) $\arcsin(x^2)$

f) $\arcsin(\arcsin(x))$ g) $\sin(\tan(x))$ h) $\tan(\sin(x))$ i) $\arcsin(\tan(x))$ j) $\arcsin(\arctan(x))$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \cos x \, dx$$

$$\arcsin(y) = \sin(x) + C$$

$$y = \sin(\sin(x) + C)$$

$$0 = \sin(\sin(0) + C)$$

$$0 = \sin(C)$$

$$0 = C$$

$$\boxed{y = \sin(\sin(x))}$$

8. **Consider the following three statements about a series $\sum_{n=1}^{\infty} a_n$ with positive terms:**

I: The series converges because $\lim_{n \to \infty} a_n = 0$.

II: The series converges because $\lim_{n \to \infty} \frac{a_{n+1}}{b_n} = 1.1$ and $\sum_{n=1}^{\infty} b_n$ converges.

III: The series converges because $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 1$.

For each statement, determine whether the reasoning is correct or incorrect.

a) I: correct, II: correct, III: correct
b) I: correct, II: correct, III: incorrect
c) I: correct, II: incorrect, III: correct
d) I: correct, II: incorrect, III: incorrect
e) I: incorrect, II: correct, III: correct
f) I: incorrect, II: correct, III: incorrect
g) I: incorrect, II: incorrect, III: correct
h) I: incorrect, II: incorrect, III: incorrect
i) Wrong answer
j) Bonus wrong answer

I incorrect – counterexample is $\sum_{n=1}^{\infty} \frac{1}{n}$

II correct – limit comparison test shows $\sum_{n=1}^{\infty} a_{n+1}$ converges and $\sum_{n=1}^{\infty} a_n = a_1 + \sum_{n=1}^{\infty} a_{n+1}$

III incorrect – ratio test inconclusive

9. **Consider the following three statements about a series $\sum_{n=1}^{\infty} a_n$ with positive terms:**

I: The series converges because $a_n < \dfrac{1}{10+\sqrt{n}}$.

II: The series diverges because $\dfrac{1}{n^2} < a_n$.

III: The series converges because $\lim_{n \to \infty} \dfrac{a_{n+1}}{a_n} = 0$.

For each statement, determine whether the reasoning is correct (C) or incorrect (F).

a) I: C, II: C, III: C
b) I: C, II: C, III: F
c) I: C, II: F, III: C
d) I: C, II: F, III: F
e) I: F, II: C, III: C
f) I: F, II: C, III: F
g) I: F, II: F, III: C
h) I: F, II: F, III: F
i) Wrong answer
j) Bonus wrong answer

I False – $\sum_{n=1}^{\infty} \frac{1}{10+\sqrt{n}}$ diverges, so comparison test yields nothing

II False – $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges so comparison test yields nothing

III Correct – ratio test

10. Consider the three series

$$\text{I: } \sum_{n=0}^{\infty} \frac{n^5}{3^n}, \qquad \text{II: } \sum_{n=0}^{\infty} \frac{10^n}{\sqrt{n!}}, \qquad \text{and} \qquad \text{III: } \sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

and the statements

(C) The series converges
(D) The series diverges

For each series, decide which of statements (C), (D) is correct.

a) I: C, II: C, III: C
b) I: C, II: C, III: D
c) I: C, II: D, III: C
d) I: C, II: D, III: D
e) I: D, II: C, III: C
f) I: D, II: C, III: D
g) I: D, II: D, III: C
h) I: D, II: D, III: D
i) Wrong answer
j) Bonus wrong answer

I Converges - root test $\lim_{n\to\infty}\left(\frac{n^5}{3^n}\right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^{\frac{5}{n}}}{3} = \frac{1}{3} < 1$

II Converges - ratio test $\lim_{n\to\infty} \frac{10^{n+1}}{\sqrt{(n+1)!}} \cdot \frac{\sqrt{n!}}{10^n} = \lim_{n\to\infty} \frac{10}{\sqrt{n+1}} = 0 < 1$

III Diverges - integral test $u = \ln x$, $du = \frac{1}{x}dx$

$$\int_2^\infty \frac{1}{x\ln(x)}\,dx = \int_{\ln(2)}^{\infty} \frac{1}{u}\,du = \ln u \Big|_{\ln(2)}^{\infty} = \infty$$

11. **Consider the two series**

$$\text{I: } \sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{1+n}\right)^n \quad \text{and} \quad \text{II: } \sum_{n=0}^{\infty} \frac{(-1)^n n^{\left(\frac{2}{3}\right)}}{1+n^{\left(\frac{4}{3}\right)}}$$

and the statements

(AC) The series converges absolutely
(CC) The series converges conditionally
(D) The series diverges

For each series, decide which of statements (AC), (CC), (D) is correct.

a) I: AC, II: AC
b) I: AC, II: CC
c) I: AC, II: D
d) I: CC, II: AC
e) I: CC, II: CC
f) I: CC, II: D
g) I: D, II: AC
h) I: D, II: CC
i) I: D, II: D
j) Wrong answer

I Diverges - Divergence test $\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n} = \lim_{n\to\infty} \left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = e^{-1} \neq 0$

II Converges - AST $\lim_{n\to\infty} \frac{n^{2/3}}{1+n^{4/3}} = 0$

$\sum_{n=0}^{\infty} \left|\frac{(-1)^n n^{2/3}}{1+n^{4/3}}\right| = \sum_{n=0}^{\infty} \frac{n^{2/3}}{1+n^{4/3}}$ diverges by LCT with $\sum_{n=0}^{\infty} \frac{1}{n^{2/3}}$

$\lim_{n\to\infty} \frac{\frac{n^{2/3}}{1+n^{4/3}}}{\frac{1}{n^{2/3}}} = \lim_{n\to\infty} \frac{n^{4/3}}{1+n^{4/3}} = 1$

So Conditionally convergent.

12. Consider the two series

$$\text{I: } \sum_{n=0}^{\infty} \frac{1}{n^{\pi}} \quad \text{and} \quad \text{II: } \sum_{n=0}^{\infty} \frac{n!}{10^{(100\,n)}}$$

and the statements

(C) The Ratio Test establishes convergence
(D) The Ratio Test establishes divergence
(F) The Ratio Test is not conclusive.

Apply the Ratio Test to series I and II and for each, decide which of statements (C), (D), (F) is correct.

a) I: C, II: C
b) I: C, II: D
c) I: C, II: F
d) I: D, II: C
e) I: D, II: D
f) I: D, II: F
g) I: F, II: C
h) I: F, II: D
i) I: F, II: F
j) Wrong answer

$$\text{I} \quad \lim_{n\to\infty} \frac{\frac{1}{(n+1)^{\pi}}}{\frac{1}{n^{\pi}}} = \lim_{n\to\infty} \frac{n^{\pi}}{(n+1)^{\pi}} = \left(\lim_{n\to\infty} \frac{n}{n+1}\right)^{\pi} = 1^{\pi} = 1$$

Inconclusive

$$\text{II} \quad \lim_{n\to\infty} \frac{(n+1)!}{10^{100(n+1)}} \cdot \frac{10^{100n}}{n!} = \lim_{n\to\infty} \frac{n+1}{10^{100}} = \infty$$

Diverges

13. Consider the two series

$$\text{I: } \sum_{n=0}^{\infty} \left(\frac{1+n^3}{10+100\,n^2+n^3} \right)^n \qquad \text{and} \qquad \text{II: } \sum_{n=1}^{\infty} \left(\frac{3+n}{3\,n} \right)^n$$

and the statements

(C) The Root Test establishes convergence
(D) The Root Test establishes divergence
(F) The Root Test is not conclusive.

Apply the Root Test to series I and II and for each, decide which of statements (C), (D), (F) is correct.

a) I: C, II: C
b) I: C, II: D
c) I: C, II: F
d) I: D, II: C
e) I: D, II: D
f) I: D, II: F
g) I: F, II: C
h) I: F, II: D
i) I: F, II: F
j) Wrong answer

I $\lim_{n\to\infty} \left(\left(\frac{1+n^3}{10+100n^2+n^3} \right)^n \right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3+1}{n^3+100n^2+10} = 1$

In conclusive

II $\lim_{n\to\infty} \left(\left(\frac{3+n}{3n} \right)^n \right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{n+3}{3n} = \frac{1}{3} < 1$

Converge

14. Let $f(x) = \frac{1}{72}x^3 e^{(2x^2)}$ **. What is** $f^{(7)}(0)$**?**

a) 20	**b) 40**	**c) 60**	**d) 80**	**e) 100**
f) 120	**g) 140**	**h) 160**	**i) 180**	**j) 200**

$f(x) = \frac{1}{72} x^3 \cdot \sum_{n=0}^{\infty} \frac{(2x^2)^n}{n!}$

$\frac{f^{(7)}(0)}{7!} = \frac{1}{72} \cdot \frac{2^2}{2!}$

$f^{(7)}(0) = \frac{7!}{36} = 7 \cdot 5 \cdot 4 = \boxed{140}$

15. Calculate $L = \lim_{x \to 0} \dfrac{120 \sin(2x^5)}{x\cos(5x^2) - x}$ by finding the Maclaurin series of the numerator and the Maclaurin series of the denominator. These two Maclaurin series begin with the same degree p monomial. (In other words, for the Maclaurin series for both the numerator and denominator, the coefficients of x^n are 0 for $n < p$ and the coefficients of x^p are nonzero.) What is the value of the product $p\,L$?

a) - 36 b) - 48 c) - 60 d) - 72 e) - 84

f) - 96 g) -108 h) -120 i) - 132 j) - 144

$$120\sin(2x^5) = 120 \sum_{n=0}^{\infty} \frac{(-1)^n (2x^5)^{2n+1}}{(2n+1)!} = 120(2x^5) - \frac{120(4x^{15})}{3!} + \dots$$

$$x\cos(5x^2) - x = -x + x \sum_{n=0}^{\infty} \frac{(-1)^n (5x^2)^{2n}}{(2n)!} = -x + \sum_{n=0}^{\infty} \frac{(-1)^n (5x^2)\cdot x}{(2n)!}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n (5x^2)^{2n}\cdot x}{(2n)!}$$

$$= -\frac{25x^5}{2!} + \frac{625x^9}{4!} - \dots$$

So $p = 5$

$$L = \lim_{x\to 0} \frac{120 \sin(2x^5)}{x\cos(5x^2 - x} = \frac{120 \cdot 2}{-\frac{25}{2}} = -\frac{120 \cdot 4}{25}$$

$$L \cdot p = 5 \cdot \frac{-480}{25} = \boxed{-96}$$

16. Calculate the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{\sqrt{n+1}\ 4^n}$. Let R be the radius of convergence. You will need to calculate the sum of four integers and it might help to record them as you go.

Let c be the base point of the power series. (c = -3)
Set $\rho = R$ if R is an integer and -1 otherwise. (ρ = 4)
Set $\sigma = 1$ if the left endpoint belongs to the interval of convergence and 0 otherwise. (σ = 0)
Set $\tau = 3$ if the right endpoint belongs to the interval of convergence and 0 otherwise. (τ = 3)

What is the value of $c + \rho + \sigma + \tau$?

a) -4	b) -3	c) -2	d) 2	e) 3
f) 4	g) 7	h) 8	i) 10	j) 11

$\frac{1}{R} = \lim_{n\to\infty} \left(\frac{1}{\sqrt{n+1}\ 4^n}\right)^{\frac{1}{n}} = \frac{1}{4(n+1)^{\frac{1}{2n}}} = \frac{1}{4}$ $R = 4$

$c - R = -3 - 4 = -7$

$\sum_{n=0}^{\infty} \frac{(-1)^n(-4)^n}{\sqrt{n+1}\ 4^n} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}}$ Div

$c + R = -3 + 4 = 1$

$\sum_{n=0}^{\infty} \frac{(-1)^n (4^n)}{\sqrt{n+1}\ 4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ Conv

$-3 + 4 + 0 + 3 = \boxed{4}$

17. Let $T(x)$ be the degree 2 Taylor polynomial of $\ln(x)$ with base point 2. What is $T(3)-\ln(2)$?

a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{3}{8}$ d) $\frac{1}{2}$ e) $\frac{5}{8}$

f) $\frac{3}{4}$ g) $\frac{7}{8}$ h) 1 i) $\frac{5}{4}$ j) $\frac{3}{2}$

$$T(x) = f(2) + \frac{f'(2)}{1!}(x-2) + \frac{f''(2)}{2!}(x-2)^2$$

$$= \ln(2) + \frac{1}{2}(x-2) + \frac{\left(-\frac{1}{4}\right)}{2}(x-2)^2$$

$$T(3) - \ln(2) = \frac{1}{2}(1) - \frac{1}{8}(1)^2 = \boxed{\frac{3}{8}}$$

18. To approximate $\displaystyle\int_0^{\frac{1}{2}} \frac{\arctan(x) - x}{x^2}\,dx$, the Maclaurin series of $\arctan(x)$ (and, from that, the Maclaurin series of the integrand) is used. An alternating series for the (exact) value S of the definite integral results. An approximation to S is obtained by using the minimum number of terms that, by the Alternating Series Test, guarantee an absolute error less than 0.001. What is the approximation?

a) $-\dfrac{96}{2401}$ b) $-\dfrac{209}{5376}$ c) $-\dfrac{19}{480}$ d) $-\dfrac{13}{336}$ e) $-\dfrac{2089}{53760}$

f) $-\dfrac{25069}{645120}$ g) $-\dfrac{131}{3360}$ h) $-\dfrac{523}{13440}$ i) $-\dfrac{3}{80}$ j) $-\dfrac{37}{960}$

$$\int_0^{\frac{1}{2}} \frac{1}{x^2}\left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} - x\right)dx$$

$$= \int_0^{\frac{1}{2}} \frac{1}{x^2}\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}\,dx$$

$$= \int_0^{\frac{1}{2}} \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{2n+1}\,dx$$

$$= \sum_{n=1}^{\infty} \int_0^{\frac{1}{2}} \frac{(-1)^n x^{2n-1}}{2n+1}\,dx$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}\cdot\frac{x^{2n}}{2n}\Big|_0^{\frac{1}{2}}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{2n(2n+1)}\cdot 4^{-n}$$

$$|a_{N+1}| < 10^{-3}$$

$$\frac{4^{-N-1}}{2(N+1)(2(N+1)+1)} < 10^{-3}$$

$$\frac{4^{-2}}{2(2)(2\cdot 2+1)} = \frac{1}{16\cdot 4\cdot 5} = \frac{1}{320} > \frac{1}{1000}$$

$$\frac{4^{-3}}{2(3)(2\cdot 3+1)} = \frac{1}{64\cdot 2\cdot 3\cdot 7} = \frac{1}{128\cdot 21} = \frac{1}{2688} < \frac{1}{1000}$$

So $N = 2$

$$\sum_{n=1}^{2} \frac{(-1)^n\cdot 4^{-n}}{2n(2n+1)} = -\frac{1}{4(2)(3)} + \frac{1}{16(4)(5)}$$

$$= -\frac{1}{24} + \frac{1}{320}$$

$$= -\frac{40}{960} + \frac{3}{960}$$

$$= \boxed{-\frac{37}{960}}$$

$320 = 32\cdot 10$

$24 = 8\cdot 3$

19. **What is the coefficient of x^5 in the Maclaurin series of $\dfrac{8x}{4-x^2}$?**

a) $\frac{1}{16}$ b) $\frac{-1}{16}$ c) $\frac{1}{8}$ d) $-\frac{1}{8}$ e) $\frac{1}{4}$

f) $-\frac{1}{4}$ g) $\frac{1}{2}$ h) $-\frac{1}{2}$ i) 2 j) -2

$$\frac{8x}{4-x^2} = 2x \cdot \frac{1}{1-\frac{x^2}{4}} = 2x \cdot \sum_{n=0}^{\infty} \left(\frac{x^2}{4}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{2x^{2n+1}}{4^n}$$

$$n = 2 \Rightarrow \frac{2x^5}{4^2} = \boxed{\frac{1}{8}}$$

20. **What is the coefficient of x^4 in the Maclaurin series of** $\dfrac{1}{(1+x^2)^{\left(\frac{1}{3}\right)}}$ **?**

a) $-\frac{1}{9}$ b) $\frac{1}{9}$ c) $-\frac{1}{6}$ d) $\frac{1}{6}$ e) $-\frac{2}{9}$

f) $\frac{2}{9}$ g) $-\frac{1}{3}$ h) $\frac{1}{3}$ i) $-\frac{2}{3}$ j) $\frac{2}{3}$

Let $f(x) = (x+1)^{-1/3}$

$f'(x) = -\frac{1}{3}(x+1)^{-4/3}$

$f''(x) = \frac{4}{9}(x+1)^{-7/3}$

$f(x) = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \ldots$

$(x^2+1)^{-1/3}$ $f(x^2) = 1 - \frac{1}{3}x^2 + \frac{2}{9}x^4 - \ldots$

$\boxed{\frac{2}{9}}$