

Math 132 Quiz
8 AM - 9 AM

NAME: _____

1. Is the series

$$\sum_{n=2}^{\infty} (-1)^n \frac{2^n}{n^{1/3} \cdot 3^n}$$

absolutely convergent, is it conditionally convergent, or is it divergent?

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \frac{2^{n+1}}{(n+1)^{1/3} 3^{n+1}} \cdot \frac{n^{1/3} 3^n}{2^n} \\ &= \lim_{n \rightarrow \infty} \frac{2}{3} \cdot \frac{n^{1/3}}{(n+1)^{1/3}} = \frac{2}{3} < 1 \end{aligned}$$

Absolutely convergent by ratio test.

2. What is the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{2^n (-1)^n (x+3)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{2^n (-3-x)^n}{\sqrt{n}}?$$

$$\frac{1}{R} = \lim_{n \rightarrow \infty} |a_n|^{1/n} = \lim_{n \rightarrow \infty} \left(\frac{2^n}{\sqrt{n}} \right)^{1/n} = \lim_{n \rightarrow \infty} \frac{2}{n^{1/2n}} = 2 \quad \Rightarrow R = \frac{1}{2}$$

$$\text{Left endpoint: } x = -3 - \frac{1}{2} \quad \sum_{n=1}^{\infty} \frac{2^n (-1)^n \left(-\frac{1}{2}\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \quad \text{DIV}$$

$$\text{Right endpoint: } x = -3 + \frac{1}{2} \quad \sum_{n=1}^{\infty} \frac{2^n (-1)^n \left(\frac{1}{2}\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \quad \text{CONV}$$

$$\left[-3 - \frac{1}{2}, -3 + \frac{1}{2}\right] = \left[-\frac{7}{2}, -\frac{5}{2}\right]$$