

Math 132 Quiz
9 AM - 10 AM

NAME: _____

1. Is the series

$$\sum_{n=2}^{\infty} (-1)^n \frac{3^{2n}}{10^n \cdot \sqrt{n^3+1}}$$

absolutely convergent, is it conditionally convergent, or is it divergent?

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \frac{3^{2(n+1)}}{10^{n+1} \sqrt{(n+1)^3+1}} \cdot \frac{10^n \sqrt{n^3+1}}{3^{2n}} \\ &= \frac{3^2}{10} \lim_{n \rightarrow \infty} \sqrt{\frac{n^3+1}{(n+1)^3+1}} = \frac{9}{10} < 1 \end{aligned}$$

Absolutely convergent by ratio test

2. What is the interval of convergence of

$$\sum_{n=2}^{\infty} \frac{2^n (-1)^n (x+5)^n}{3 \ln(n)} = \sum_{n=2}^{\infty} \frac{2^n (-5-x)^n}{\ln(n^3)}?$$

$$\frac{1}{R} = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{2^{n+1}}{3 \ln(n+1)} \cdot \frac{3 \ln(n)}{2^n} = 2 \lim_{n \rightarrow \infty} \frac{\ln(n)}{\ln(n+1)} = 2 \Rightarrow R = \frac{1}{2}$$

$$\text{Left endpoint: } x = -5 - \frac{1}{2} \quad \sum_{n=2}^{\infty} \frac{2^n (-1)^n \left(-\frac{1}{2}\right)^n}{3 \ln(n)} = \sum_{n=2}^{\infty} \frac{1}{3 \ln(n)} \quad \text{DIV}$$

$$\text{Right endpoint: } x = -5 + \frac{1}{2} \quad \sum_{n=2}^{\infty} \frac{2^n (-1)^n \left(\frac{1}{2}\right)^n}{3 \ln(n)} = \sum_{n=2}^{\infty} \frac{(-1)^n}{3 \ln(n)} \quad \text{CONV}$$

$$\left(-5 - \frac{1}{2}, -5 + \frac{1}{2}\right) = \left[-\frac{11}{2}, -\frac{9}{2}\right]$$