Math 132 Quiz
Noon - 1 PM

NAME:________________

1. Is the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{3^{3n}}{7^{2n} \cdot (n^{1/2}+1)}$$

absolutely convergent, is it conditionally convergent, or is it divergent?

$$\lim_{n\to\infty} |a_n|^{1/n} = \lim_{n\to\infty} \left(\frac{3^{3n}}{7^{2n}(n^{1/2}+1)}\right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{3^3}{7^2(n^{1/2}+1)^{1/n}}$$

$$= \frac{27}{49} \lim_{n\to\infty} (n^{1/2}+1)^{-1/n} = \frac{27}{49} < 1$$

Absolutely convergent by root test

2. What is the interval of convergence of

$$\sum_{n=0}^{\infty} \frac{3^n(-1)^n(x+3)^n}{\sqrt{1+n^{2/3}}} = \sum_{n=0}^{\infty} \frac{3^n\,(-3-x)^n}{\sqrt{1+n^{2/3}}}?$$

$c=-3$

$$\frac{1}{R} = \lim_{n\to\infty} |a_n|^{1/n} = \lim_{n\to\infty} \left(\frac{3^n}{\sqrt{1+n^{2/3}}}\right)^{\frac{1}{n}} = 3 \lim_{n\to\infty} (1+n^{2/3})^{-\frac{1}{2n}} = 3 \quad \Rightarrow R = \frac{1}{3}$$

Left endpoint: $x = -3 - \frac{1}{3}$ $\quad \sum_{n=0}^{\infty} \frac{3^n(-1)^n(-\frac{1}{3})^n}{\sqrt{1+n^{2/3}}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{1+n^{2/3}}}$ DIV

Right endpoint: $x = -3 + \frac{1}{3}$ $\quad \sum_{n=0}^{\infty} \frac{3^n(-1)^n(\frac{1}{3})^n}{\sqrt{1+n^{2/3}}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{1+n^{2/3}}}$ CONV

$$\left(-3-\frac{1}{3}, -3+\frac{1}{3}\right] = \left(-\frac{10}{3}, -\frac{8}{3}\right]$$