

Math 132 Quiz
9 AM - 10 AM

1. Find the arc length of the graph of $y = x^2 - \ln(x)/8$, $1 \leq x \leq 2$.

$$\begin{aligned} s &= \int_1^2 \sqrt{1 + (f'(x))^2} dx \\ &= \int_1^2 \sqrt{1 + \left(2x - \frac{1}{8x}\right)^2} dx \\ &= \int_1^2 \sqrt{1 + 4x^2 - \frac{1}{2} + \frac{1}{64x^2}} dx \\ &= \int_1^2 \sqrt{4x^2 + \frac{1}{2} + \frac{1}{64x^2}} dx \\ &= \int_1^2 \sqrt{\left(2x + \frac{1}{8x}\right)^2} dx \\ &= \int_1^2 \left(2x + \frac{1}{8x}\right) dx \\ &= \left. x^2 + \frac{\ln x}{8} \right|_1^2 \\ &= \left(4 + \frac{\ln 2}{8}\right) - (1) = 3 + \frac{\ln 2}{8} \end{aligned}$$

2. If $f(x) = 4x^3/65$, $2 \leq x \leq 3$ is the probability density function of a random variable X , what is the mean μ_X ?

$$\begin{aligned} \mu &= \int_2^3 x f(x) dx \\ &= \int_2^3 \frac{4x^4}{65} dx \\ &= \frac{4}{5 \cdot 65} x^5 \Big|_2^3 \\ &= \frac{4}{325} (243 - 32) \\ &= \frac{4}{325} \cdot 211 \\ &= \frac{844}{325} \end{aligned}$$