

Math 132 Quiz
8 AM - 9 AM

1. Is the series

$$\sum_{n=1}^{\infty} \frac{2^{2n} + 3^n}{n2^{2n}}$$

convergent or is it divergent? Justify

Comparison test:

$$a_n = \frac{2^{2n} + 3^n}{n2^{2n}} = \frac{1}{n} \left(\frac{2^{2n} + 3^n}{2^{2n}} \right) = \frac{1}{n} \left(1 + \frac{3^n}{2^{2n}} \right) > \frac{1}{n}$$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges and $a_n > \frac{1}{n}$, $\sum_{n=1}^{\infty} \frac{2^{2n} + 3^n}{n \cdot 2^{2n}}$ diverges

2. Let

$$S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3} \quad \text{and} \quad S_N = \sum_{n=1}^N (-1)^n \frac{1}{n^3}.$$

For what N can you be sure that $|S - S_N| < 0.001$?

$$\text{Need } |a_{N+1}| \leq \frac{1}{1000} = 0.001$$

$$\frac{1}{(N+1)^3} \leq \frac{1}{1000}$$

$$1000 \leq (N+1)^3$$

$$10 \leq N+1$$

$$9 \leq N$$

$$\text{With } N=9, \quad |S - S_N| \leq |a_{N+1}| = a_{10} = \frac{1}{1000}$$