Math 132 Quiz
12 Noon - 1 PM

1. Is the series

$$\sum_{n=1}^{\infty} \frac{n2^n + \ln(n)}{n3^n + 1}$$

convergent or is it divergent?. $\underline{\text{Justify}}$

Limit Comparison test:

$$\lim_{n\to\infty} \frac{a_n}{(\frac{2}{3})^n} = \lim_{n\to\infty} \frac{n\cdot 2^n + \ln(n)}{n\cdot 3^n + 1}\cdot\frac{3^n}{2^n} = \lim_{n\to\infty}\frac{n\cdot 3^n}{n\cdot 3^n+1} + \lim_{n\to\infty}\frac{3^n \ln(n)}{2^n(n\cdot 3^n+1)}$$

$$= 1 + \left(\lim_{n\to\infty}\frac{\ln(n)}{2^n}\right)\left(\lim_{n\to\infty}\frac{3^n}{n\cdot 3^n+1}\right)$$

$$= 1 + 0 = 1$$

Since $\lim_{n\to\infty}\frac{\ln(n)}{2^n} = 0$ and $\lim_{n\to\infty}\frac{3^n}{n\cdot 3^n+1} = \lim_{n\to\infty}\frac{3^n \ln(3)}{3^n + n3^n\ln(3)} = \lim_{n\to\infty}\frac{\ln(3)}{1+n\ln(3)} = 0$

So $\sum_{n=1}^{\infty}\frac{n\cdot 2^n + \ln(n)}{n\cdot 3^n+1}$ converges by LCT since $\sum_{n=1}^{\infty}(\frac{2}{3})^n$ converges.

2. Let

$$S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n^2+1} \qquad \text{and} \qquad S_N = \sum_{n=1}^{N} (-1)^n \frac{1}{2n^2+1}.$$

For what N can you be sure that $|S - S_N| < 0.01$?

Need $|a_{N+1}| \leq \frac{1}{100} = 0.01$

$$\frac{1}{2(N+1)^2+1} \leq \frac{1}{100}$$

$$100 \leq 2(N+1)^2+1$$

$$\frac{99}{2} \leq (N+1)^2$$

Since N an integer, $(N+1)^2 \geq \frac{128}{2} = 64$

$$N+1 \geq 8$$

$$N \geq 7$$