

# Exam 1

## Math 217

This exam consists of 16 questions worth 5 points each. You must show all work. Answers without work will receive no credit. Implicit solutions are acceptable where applicable.

For numbers 1-3, Consider the following four first order differential equations. One is separable, one is linear, one is exact, and one we have not discussed how to solve. Solve the three solvable ones.

$$\frac{dy}{dx} = 1 - \frac{y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{y} - \frac{y}{2x}$$

$$\frac{dy}{dx} = -\frac{y^2}{2x}$$

$$\frac{dy}{dx} = \frac{1}{y} - \frac{y^2}{2x}$$

1. Solve the separable one.

2. Solve the linear one.

3. Solve the exact one.

4. Use Euler's Method with  $h = 0.2$  to estimate  $y(3)$  in the following IVP:

$$y' + y = 3, y(2.4) = 1$$

5. Compute the Wronskian  $W(t^2e^t, te^t)$ .

6. A 200L tank is full of a 20 g/L salt solution. Pure water is pumped in at 10 L/min, and the tank is drained at 10L/min. Find an expression for the amount of salt in the tank as a function of time.

7. Solve  $(3x^2e^y + 2x(\cos y + 1))dx + (x^3e^y - x^2 \sin y + 2e^y \cos y)dy = 0$

8. Solve  $(x^2y + 3xy + 2y + x^2 + 3x + 2)dx + (x + 2)dy = 0$ .

9. Find and classify all the equilibria of the following:

$$y' = y^6 + 3y^5 - 4y^3$$

10. Solve  $y' = \frac{1}{9}(y + 2)^2(y - 1)$

11. A 2 kg mass is suspended from a spring, upon which the spring extends  $\frac{10}{13}$  m. Friction in the spring exerts 8 N of force when the mass is traveling 1 m/s. The mass is pushed up  $\frac{1}{2}$  m and released from rest. Give an expression for the motion of the mass as a function of time.

12. A 4 kg mass is suspended from a spring, upon which the spring extends 0.4 m. The system undergoes an external force of  $F(t) = 36 \cos(4t)$  and has no friction. If the mass is pulled down 0.5m and given an initial velocity of 5 m/s downwards, give an expression for the motion of the mass as a function of time.

13. Solve  $y'' - 6y' + 10y = \frac{e^{3t}}{\tan t}$

14. Solve  $y'' - 4y = 2e^t + e^{2t}$

15. Given that  $y_1 = x$  is a solution, find the general solution of  $(1 - x)y'' + xy' - y = 0$ . HINT:  $\frac{d}{dx}\left(\frac{e^x}{x}\right) = e^x\left(\frac{x-1}{x^2}\right)$

16. Given that  $y_1 = e^x$  and  $y_2 = x$  are solutions of the corresponding homogeneous equation, find the general solution of

$$(1 - x)y'' + xy' - y = e^x(1 - x)^2$$