

# Exam 2

## Math 217

This exam consists of 16 questions worth 5 points each. You must show all work. Answers without work will receive no credit. Recursion relations are acceptable for series solutions.

1. Solve  $y^{(4)} + 2y''' - 2y'' + 2y' - 3y = 0$

2. Solve  $y''' + 2y'' - y' - 2y = 20 \sin(3t)$

3. Solve  $y''' + 4y'' - 7y' - 10y = 0$

4. Solve  $y''' + y' = \tan t$

5. A spring ( $k=3$ ) has one end attached to an immovable surface, and the other to the top of a 4 kg mass. Another spring ( $k=2$ ) is attached to the bottom of the first mass, and has another 4 kg mass attached to its bottom end. Find an expression for the motion of the bottom mass as a function of time.

6. Solve  $2x^2y'' + 7xy' - 3y = 0$

7. Solve  $x^2y'' + y = 0$

8. Solve  $x^2y'' - xy' + y = 0$

9. List and classify the singularities of  $x(x+3)^2(x-5)^2y'' + (x+3)y' + (x-5)y = 0$

10. Give a power series solution of  $(x-1)y'' + 3xy' - y = 0$  centered at  $x_0 = 0$ .

11. Give a power series solution of  $xy'' + 3xy' - y = 0$  centered at  $x_0 = 1$ .

12. Give a power series solution of  $xy'' - 3y' - y = 0$ ,  $x > 0$ , centered at  $x_0 = 0$ .

13. Give a nontrivial lower bound for the radius of convergence of series solutions of

$$(x^3 - x^2 + x - 1)y'' + (x - 1)y' + y = 0$$

centered at  $x_0 = -\frac{1}{2}$ .

14. Give a nontrivial lower bound for the radius of convergence of series solutions of

$$(x^2 - 4)y'' + (x^2 + x - 2)y' + (x + 2)y = 0$$

centered at  $x_0 = -1$ .

15. Show that  $\mathcal{L}\{\cos 2t\} = \frac{s}{s^2+4}$

16. Show that  $\mathcal{L}\{2t + t^2\} = \frac{2s+2}{s^3}$