Final Exam

Math 217

This exam consists of 16 questions worth 5 points each. You must show all work. Answers without work will receive no credit.

1. A 2kg mass is suspended from a spring, upon which the spring extends $\frac{10}{13}$ m. Friction in the spring exerts 8N of force when the mass is traveling 1 m/s. The mass is pushed up $\frac{1}{2}$m and released from rest. Give an expression for the motion of the mass as a function of time (use $g = 10m/s^2$).

2. A 200L tank contains a 5 g/L salt solution. A 20 g/L salt solution is pumped in at 10 L/min and the (well mixed) tank is drained at 10 L/min. Find an expression for the amount of salt in the tank as a function of time.

3. Solve $\frac{dy}{dx} = 1 - \frac{y}{2x}$, $y(1) = \frac{11}{3}$.

4. Given that $y_1 = e^x$ and $y_2 = x$ are solutions of the corresponding homogeneous equation, find the general solution of

$$(1-x)y'' + xy' - y = e^x(1-x)^2$$

5. Given that $y_1 = e^x$ is a solution, find the general solution of $(1-x)y'' + xy' - y = 0$.

6. Solve $y^{(3)} - 3y' + 2y = 10\sin t$

7. Solve $x^2y'' + 3xy' + y = 0$, $x > 0$, $y(1) = 2$, $y'(1) = 3$

8. Give a power series solution of $xy'' + 3xy' - y = 0$ centered at $x_0 = 1$

9. Give (and justify) a nontrivial lower bound for the radius of convergence of series solutions of

$$(x^3 + x^2 + x + 1)y'' + (x^2 + 1)y' + y = 0$$

centered at $x_0 = 1$.

10. Show that $\mathcal{L}\{e^{5t}\} = \frac{1}{s-5}$.

11. Show $\sin 5t * \cos 5t = \frac{1}{2} t \sin 5t$

12. Solve $y'' + y = g(t)$, $y(0) = 0$, $y'(0) = 0$ for $g(t) = \begin{cases} 0 & t \leq 1 \\ t-1 & 1 \leq t \leq 3 \\ 2 & t \geq 3 \end{cases}$

13. Solve $y'' + 4y = \delta(t - \pi) - \delta(t - 2\pi)$, $y(0) = 0$, $y'(0) = 0$.

14. Solve $\vec{x}' = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} \vec{x}$, $\vec{x}(0) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

15. List and classify the critical points of $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} x(1-x-y) \\ y(\frac{3}{4}-y-\frac{1}{2}x) \end{pmatrix}$.

16. Solve $y'' + y = 0$, $y(0) = 1$, $y(\frac{\pi}{2}) = 2$.