

Math 416 Complex variables

Problem Set 1

Due: September 10 in class

1. Compute the following.

(i) $(\sqrt{3} + i)^8$

(ii) $(1 - i)^{-16}$

2. Find all the roots, and express them in rectangular coordinates.

(i) $(-25)^{\frac{1}{4}}$

(ii) $(\sqrt{3} - i)^{\frac{1}{3}}$

3. Draw the following curves in the complex plane.

(i) $|z - 2 + i| \leq 2$

(ii) $2z - 2\bar{z} = 3$

4. Prove the triangular inequality: if z_1 and z_2 are complex numbers then

$$|z_1 + z_2| \leq |z_1| + |z_2|.$$

5. Find the principal value of the argument of $\frac{i}{-2-2i}$.

6. Show that the equation $|z - z_0| = k$ can be written as

$$|z|^2 - 2 \operatorname{Re}(z\bar{z}_0) + |z_0|^2 = k^2.$$