Math 416 Complex variables

Problem Set 5

Due: September 15 in class

1.Evaluate the following integrals.

(i) $\int_1^2 (\frac{1}{t} - i)^2 dt$

(ii) $\int_0^\infty e^{-zt} dt \qquad (\text{Re } z > 0)$

2. Show that if m and n are integers,

$$\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta = \begin{cases} 0 & \text{when } m \neq n, \\ 2\pi & \text{when } m = n. \end{cases} .$$

3. Suppose that a function $f(z)$ is analytic at a point $z_0 = z(t_0)$ lying on a smooth arc $z = z(t)(a \leq t \leq b)$. Show that if $w(t) = f(z(t))$, then

$$w'(t) = f'(z(t))z'(t).$$

at $t = t_0$.

4. In each of the following cases, use the parametric representation for C to evaluate

$$\int_C f(z)dz.$$

(i) $f(z) = \pi \exp(\pi \bar{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1 + i$, and i, oriented counterclockwise.

(ii) $f(z)$ is defined by

$$f(z) = \begin{cases} 1 & \text{when } y < 0, \\ 4y & \text{when } y > 0, \end{cases}$$

and C is the arc from $-1 - i$ to $1 + i$ along the curve $y = x^3$.

(iii) $f(z) = 1$ and C is an arbitrary contour from any fixed point z_1 to any fixed point z_2.

(iv) $f(z)$ is the branch

$$z^{-1+i} = \exp((-1+i)\log(z)) \quad (|z| > 0, 0 < \arg z < 2\pi).$$

of the indicated power function, and C is the unit circle $z = e^{i\theta}(0 \leq \theta \leq 2\pi)$.