MATH 429, LINEAR ALGEBRA

FALL 2010

1. Show that if A is a matrix, and $p(x)$ is a polynomial such that $p(A) = 0$, the the minimal polynomial of A divides $p(x)$.

2. Let V be an n-dimensional vector space, and let $T : V \to V$ be a linear transformation. Show that the characteristic and minimal polynomials of T have the same roots.

3. Let V be a finite dimensional vector space. Let $W_1, \dots, W_k$ be subspaces of V and let $W = W_1 + \cdots + W_k$. Show that the following are equivalent:

(1) $W_1, \dots, W_k$ are independent.

(2) For each j, $2 \le j \le k$, we have
$$W_j \cap (W_1 + \cdots + W_{j-1}) = 0.$$

(3) If $\mathcal{B}_i$ is an ordered basis for W_i, then $\mathcal{B} = (\mathcal{B}_1, \dots, \mathcal{B}_k)$ is an ordered basis for W.

(4) $\dim V = \dim W_1 + \dots \dim W_k$.

4. (The Gram-Schmidt Process) Let V be an inner product space, and let $\beta_1, \dots, \beta_m$ be linearly independent vectors in V. Show that there are orthogonal vectors $\alpha_1, \dots, \alpha_m$ in V such that
$$\text{Span}\{\alpha_1, \dots, \alpha_j\} = \text{Span}\{\beta_1, \dots, \beta_j\}$$
for every $1 \le j \le m$.

5. Show that if V is a finite dimensional inner space, and if W is a subspace of V,
$$V = W \oplus W^{\perp}.$$