MATH 429, LINEAR ALGEBRA

FALL 2010

1. Suppose that V is a vector space of dimension n and $\mathcal{B} = \{\alpha_1, \ldots, \alpha_n\}$ is a basis for V. If $T, U : V \to V$ are linear transformations, show that

$$[T \circ U]_{\mathcal{B}} = [T]_{\mathcal{B}} \, [U]_{\mathcal{B}}.$$

2. If W is a subspace of a vector space V, then

$$\dim W + \dim W^0 = \dim V.$$

3. Show that $\det(AB) = (\det A)(\det B)$ for any two $n \times n$ matrices A and B.

4. Show that $(\mathrm{adj} A)A = (\det A)I$ for any $n \times n$ matrix A.

5. Show that $\det A = \det A^t$.