Complex Analysis, Fall 2017

Problem Set 4

Due: October 3 in class

1. Find the linear fractional transformation which sends the circle $|z| = 2$ to the circle $|z+1| = 1$, the point -2 to 0 and the point 0 to i.

2. Compute $\int_\gamma x \, dz$ where γ is the line segment from 0 to $1+i$.

3. Find an open set over which $\sqrt{1+z} + \sqrt{1-z}$ is holomorphic.

4. Express arctan in terms of log. What is an open set over which arctan is holomorphic? Justify your answer.

5. (a) Show that if f is a linear fractional transformation which sends the real axis to the imaginary axis and 1 to ∞, then f sends every circle which passes through 1 and has a real number as it center to a line parallel to the real axis.

(b) Use the previous part to construct a conformal holomorphic bijection from the region between the two circles $|z| = 1$ and $|z - \frac{1}{2}| = \frac{1}{2}$ onto the strip $0 < y < i, x \in \mathbf{R}$.

6. (a) Show that the function $f(z) = \frac{1}{2}(z + \frac{1}{z})$ is a conformal holomorphic bijection from outside the unit circle onto the plane from which the segment $[-1, 1]$ is removed.

(b) Use part (a) and write $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$ as the composition of various functions to show that $\sin z$ maps the stip $0 < x < \frac{\pi}{2}, y < 0$ bijectively to the first quadrant $x > 0, y < 0$.