

Math 5031, Algebra I

Problem Set 1

Due: September 8 in class

1. Let G be a group such that every element of G other than the unit element has order 2. Show that G is an abelian group.

2. If H and K are subgroups of a finite group G , then show that

$$|HK| = \frac{|H||K|}{|H \cap K|}.$$

3. If H is a subgroup of a group G such that $[G : H] = 2$, then show that H is a normal subgroup of G .

4. Let G be a group and let $a, b \in G$. Then the *commutator* of a and b , denoted by $[a, b]$ is defined to be

$$[a, b] = a^{-1}b^{-1}ab.$$

The subgroup of G generated by all the commutators in G is called the *commutator subgroup* of G and is denoted by G' .

(i) Show that G' is a normal subgroup of G .

(ii) If N is a normal subgroup of G , then show that G/N is abelian if and only if G' is a subgroup of N .

5. Let G be the subgroup of $GL(2, \mathbf{C})$ which is generated by

$$A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

- (i) Show that G is of order 8 and it is not abelian.
- (ii) Show that every subgroup of G is normal.