

# Math 5031, Algebra I

## Problem Set 9

Due: November 17 in class

$R$  is always a commutative ring.

1. If  $M$  and  $N_i, i \in I$  are modules over  $R$ , show that

$$M \otimes (\oplus_{i \in I} N_i) \simeq \oplus_{i \in I} (M \otimes N_i).$$

2. Let  $I$  be an ideal of  $R$ , and let  $M$  be a module over  $R$ . Show that  $A/I \otimes_R M$  is isomorphic to  $M/IM$ .

2. Let

$$M' \rightarrow M \rightarrow M'' \rightarrow 0$$

be a sequence of  $R$ -modules. Show that if

$$0 \rightarrow \text{Hom}(M'', P) \rightarrow \text{Hom}(M, P) \rightarrow \text{Hom}(M', P)$$

is exact for every  $R$ -module  $P$ , then the sequence itself is exact.

3. Show that over a PID, a module  $M$  is projective if and only if it is free.

4. Show that

- (a) A direct sum of projective modules is projective. Is this true if we replace direct sum with direct product?
- (b) A direct product of injective modules is injective.